جـامـعـة بنهـــا

كلية الحاسبات والذكاء الإصطناعي

Calculus

Lecture 03

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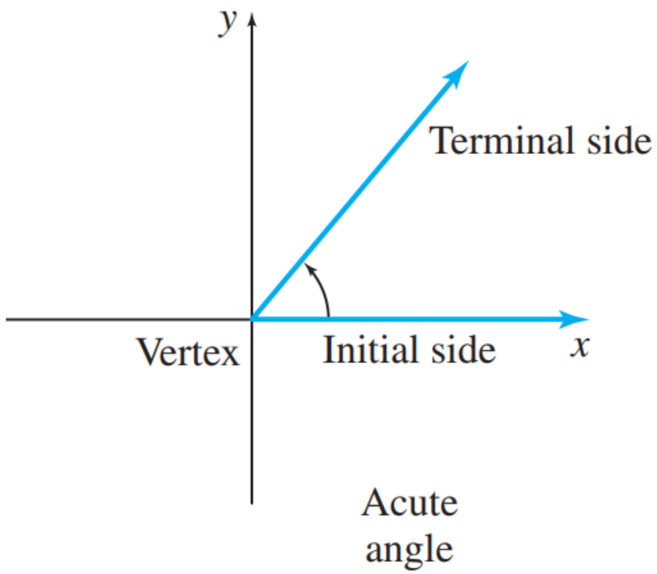
Spring 2023

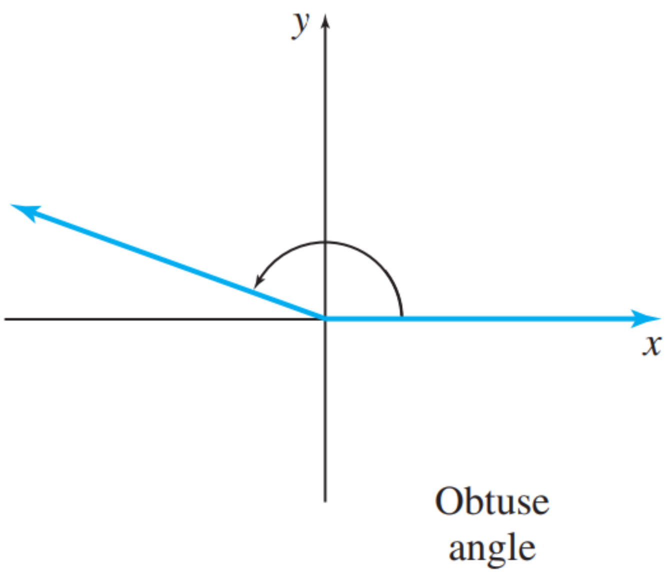
Chapter 1 Topics

- Numbers and Sets.
- Representations of Functions.
- Domain & Range of Functions.
- Algebra of Functions.
- Increasing and Decreasing.
- Test for Even and Odd Functions.
- Types of Functions and their Graphs.
- Transformations of Functions.

Angels (1/6):

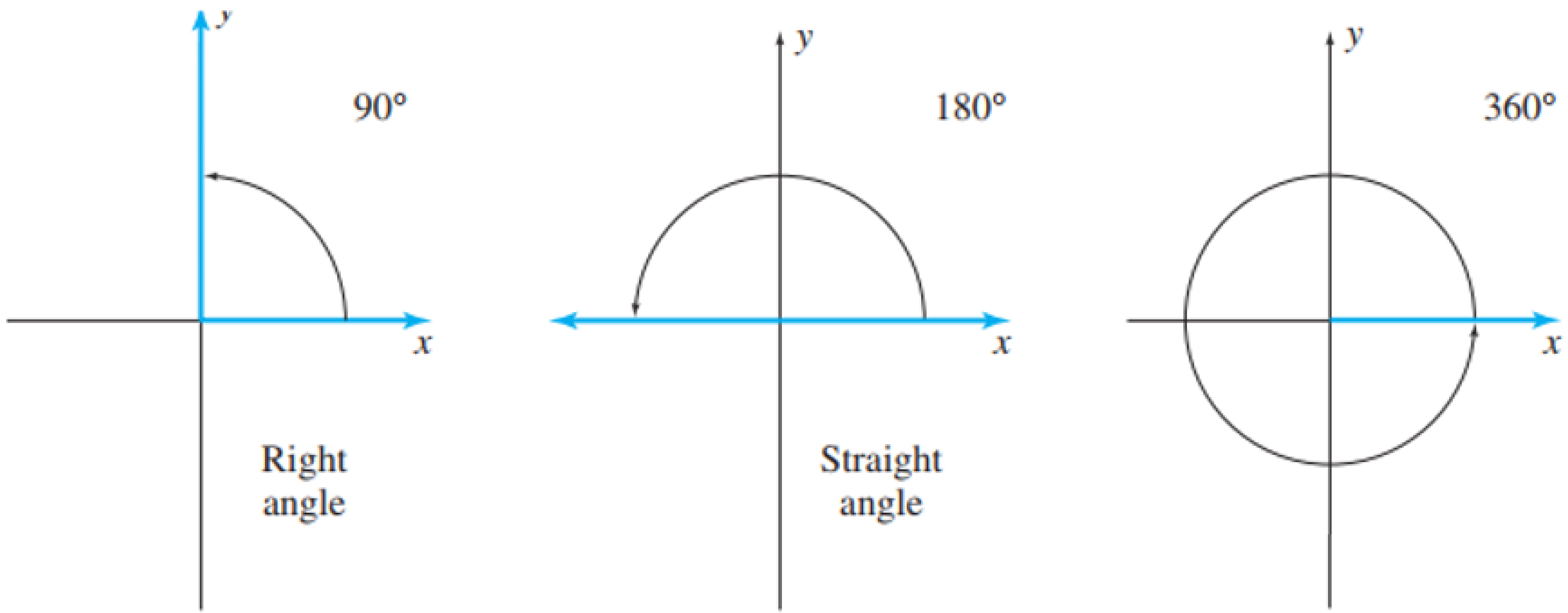
The angle is one of the basic concepts of trigonometry. It is common practice to use $\boldsymbol{\theta}$ to represent angel. Angles between 0° and 90° are **acute**, and angles between 90° and 180° are **obtuse**.





Angels (2/6):

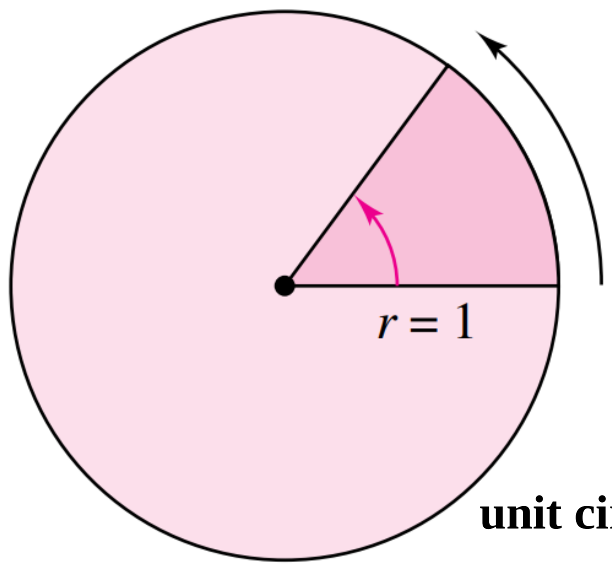
In **degree measure**, 360 degrees represents a complete rotation of a ray.



Angels (3/6):

Using degree measurement in calculus is complicated. Fortunately, there is an alternative system, called **radian measure** (abbreviated as **rad**), that helps to keep the formulas for derivatives and antiderivatives as simple as possible.

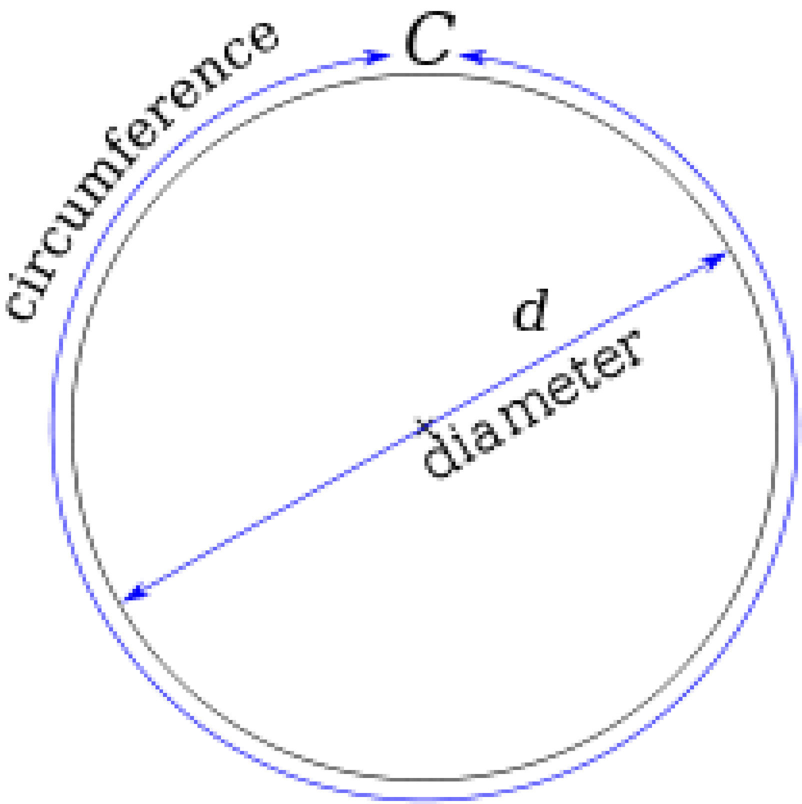
$$\pi \text{ rad} = 180^\circ$$



Angels (4/6):

π "Pi"

Derived from the first letter of the Greek word *perimetros*, meaning circumference



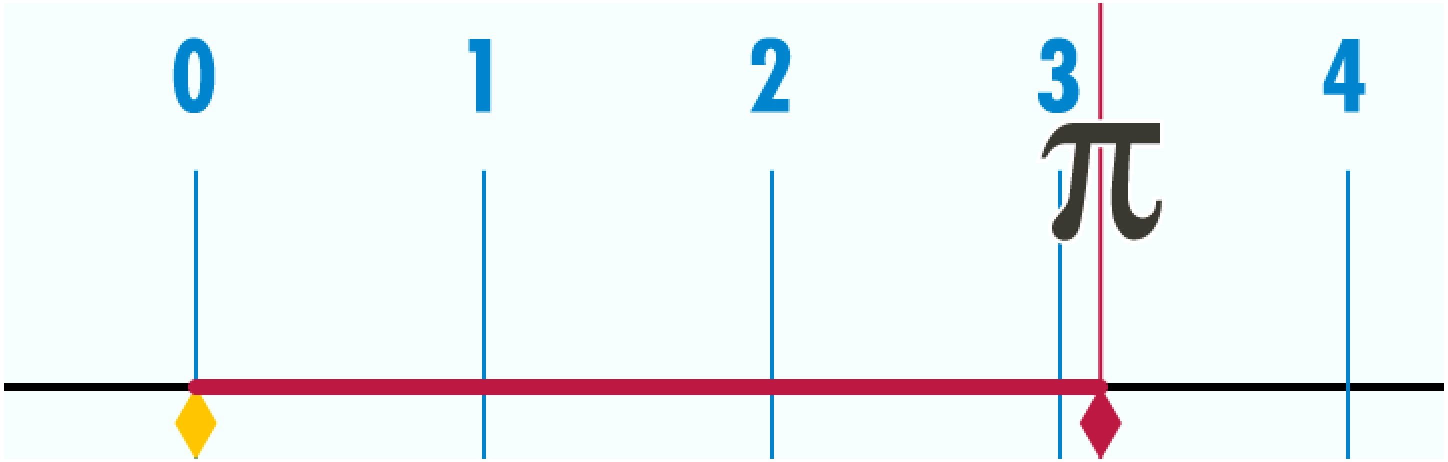
$$\pi = \frac{C}{d}$$

where $d = 2r$

Types of Fun. & Graph (14/18)

Angels (5/6):

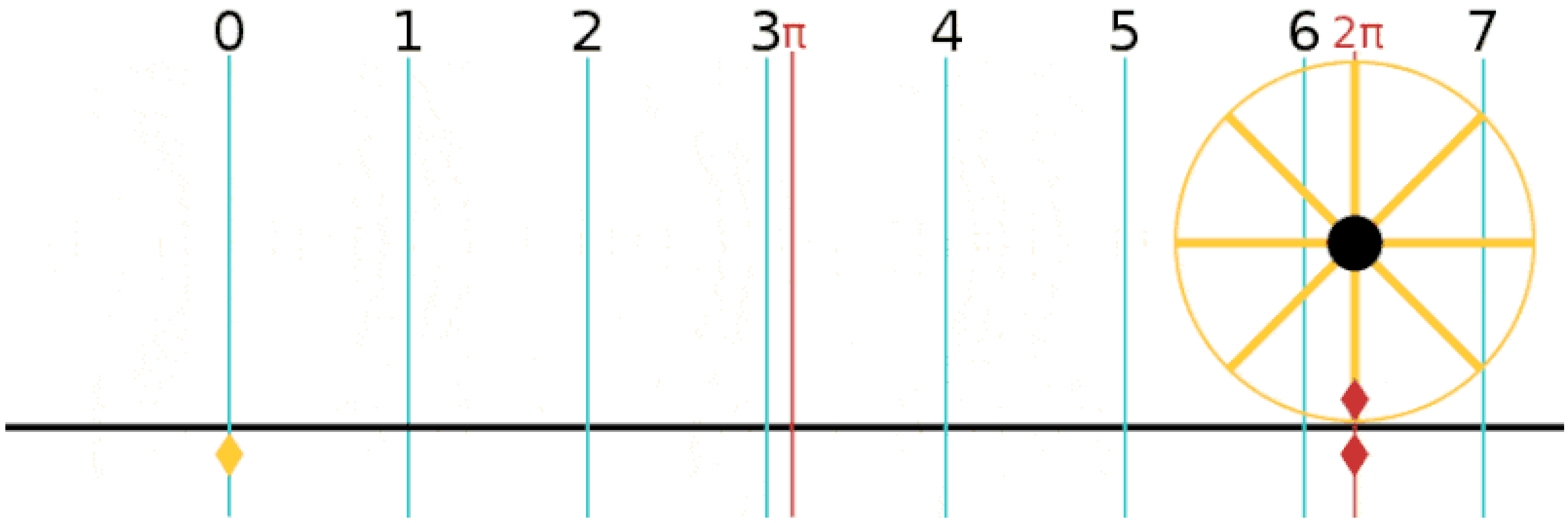
π "Pi"



When a circle's **diameter** is 1,
its circumference is π.

Angels (6/6):

Unit Circle



unit circle—its circumference is 2π.

The First Thousand Digits of π:

$\pi = 3.$

1415926535897932384626433832795028841971693993751058209749445923078164062 8
6208998628034825342117067982148086513282306647093844609550582231725359408 1
2848111745028410270193852110555964462294895493038196442881097566593344612 8
4756482337867831652712019091456485669234603486104543266482133936072602491 4
1273724587006606315588174881520920962829254091715364367892590360011330530 5
4882046652138414695194151160943305727036575959195309218611738193261179310 5
1185480744623799627495673518857527248912279381830119491298336733624406566 4
3086021394946395224737190702179860943702770539217176293176752384674818467 6
6940513200056812714526356082778577134275778960917363717872146844090122495 3
4301465495853710507922796892589235420199561121290219608640344181598136297 7
4771309960518707211349999998372978049951059731732816096318595024459455346 9
0830264252230825334468503526193118817101000313783875288658753320838142061 7
1776691473035982534904287554687311595628638823537875937519577818577805321 7
12268066130019278766111959092164201 98

Types of Fun. & Graph (16/18)

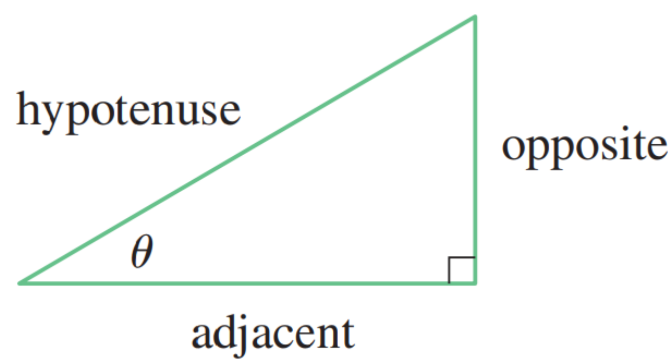
Trigonometric (1/16):

For an acute angle θ, the six trigonometric functions are sine, cosine, tangent, cotangent, secant, and cosecant (abbreviated as **sin**, **cos**, **tan**, **cot**, **sec**, and **csc**, respectively):

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$



Trigonometric (2/16):

A trigonometric identity is a relationship among the trigonometric functions.

Elementary Trigonometric Identities

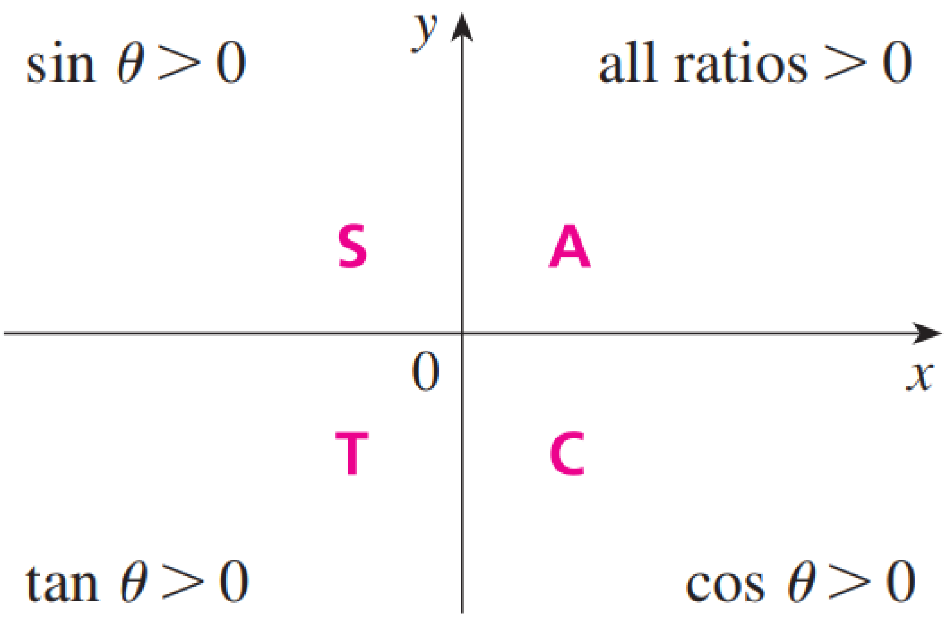
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

These identities are meaningless when the denominator is zero.

Trigonometric (3/16):

The signs of the trigonometric functions for angles in each of the four quadrants can be remembered by means of the rule "**A**ll **S**tudents **T**ake **C**alculus"

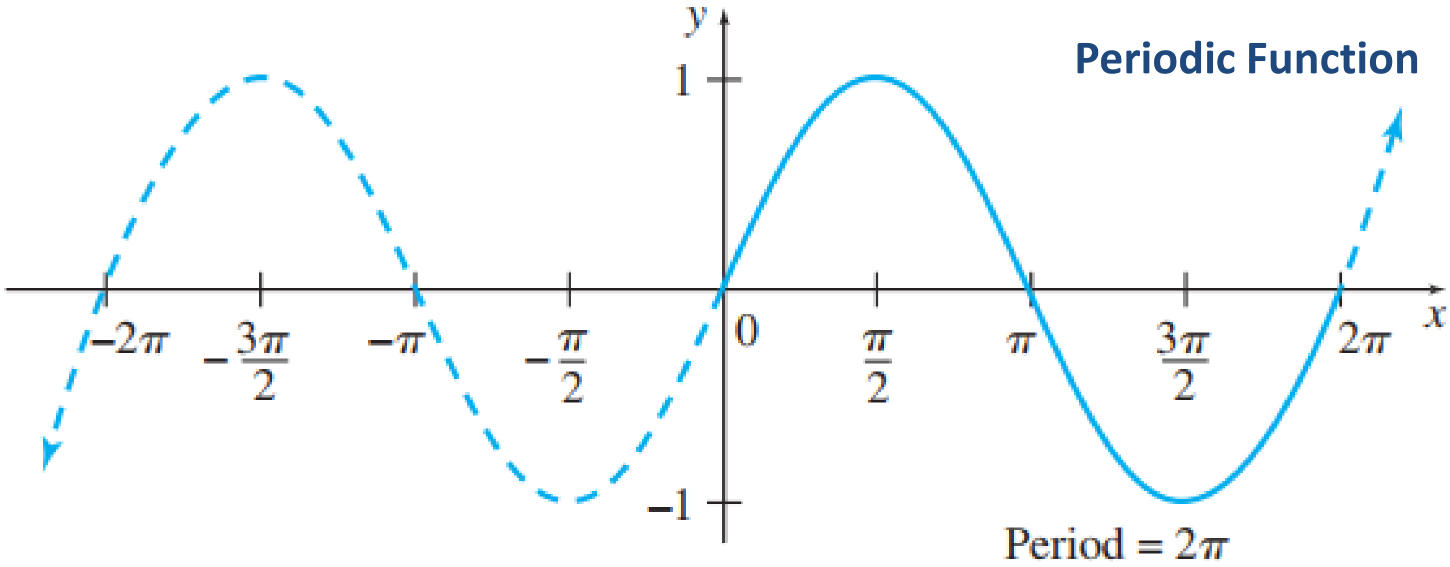


Types of Fun. & Graph (16/18)

Trigonometric (4/16):

$$f(x) = \sin x$$

Values of the Sine Function									
x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin x$	0	0.7	1	0.7	0	−0.7	−1	−0.7	0



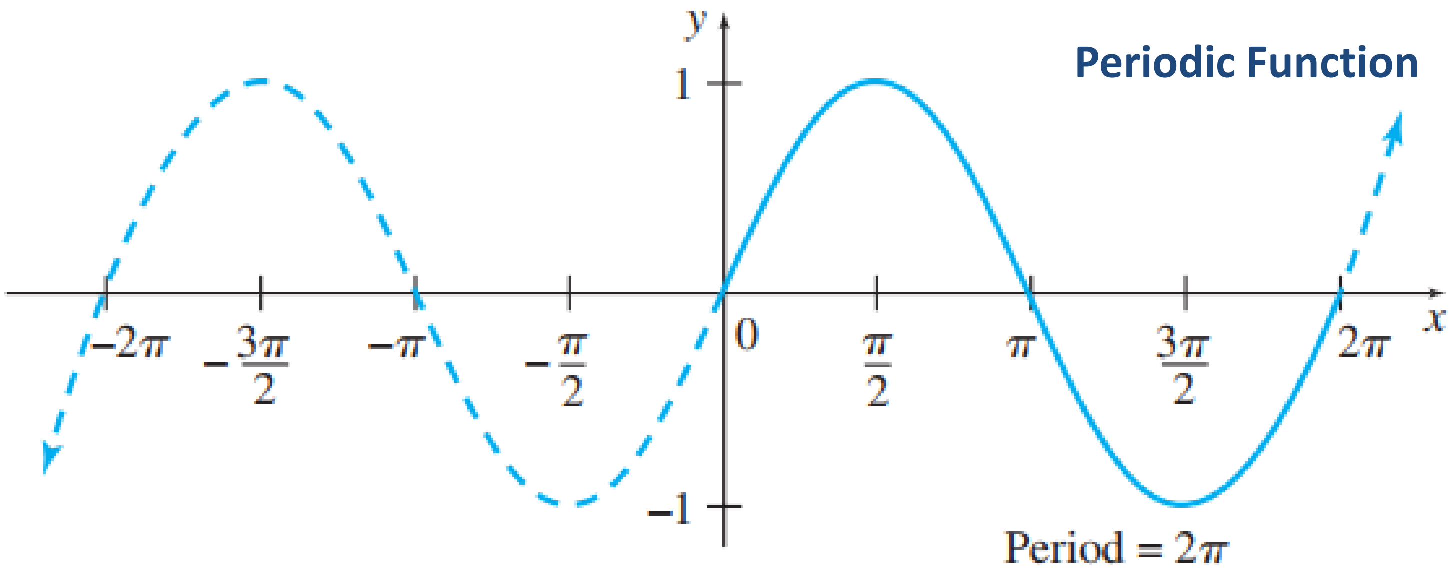
Periodic Function

Trigonometric (4/16):

Note that: $\sin(x \pm 2\pi) = \sin(x)$

$$f(x) = \sin x$$

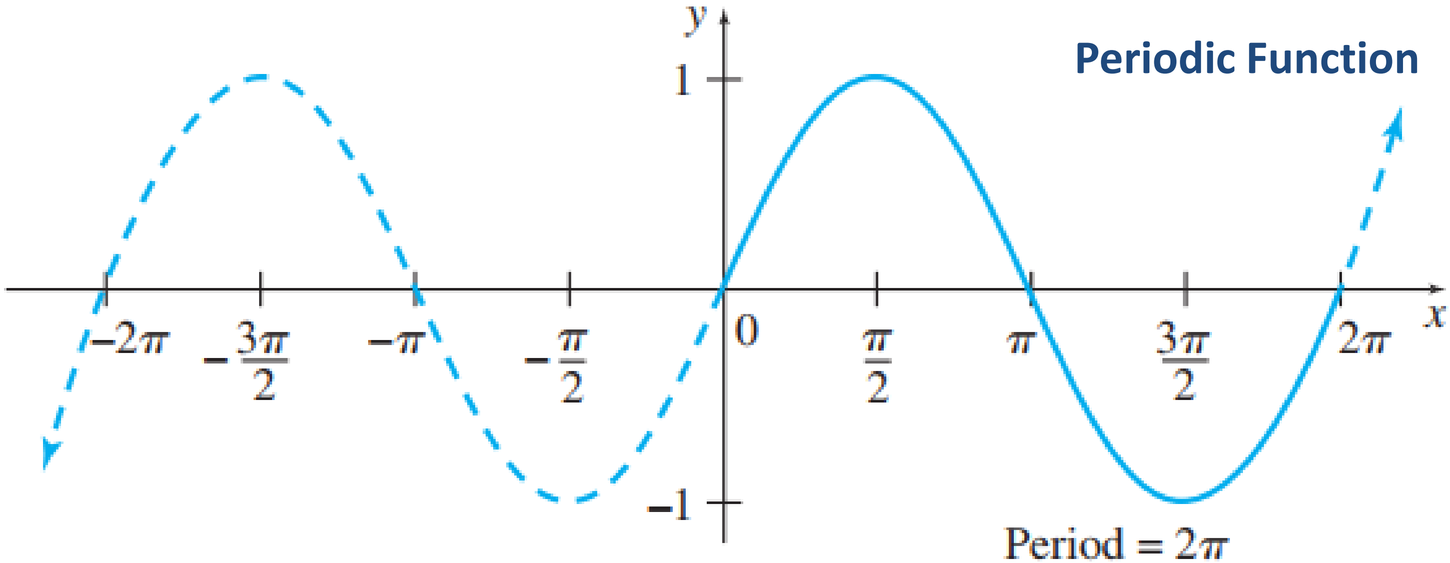
Values of the Sine Function									
x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin x$	0	0.7	1	0.7	0	−0.7	−1	−0.7	0



Types of Fun. & Graph (16/18)

Trigonometric (5/16):

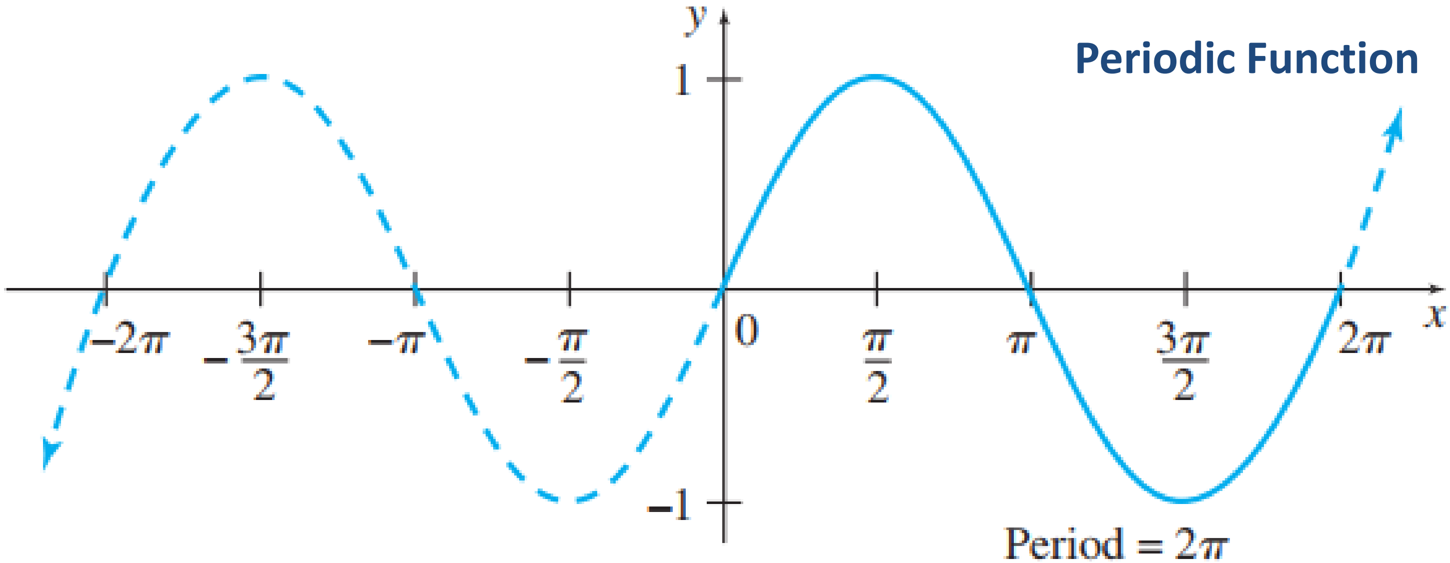
$f(x) = \sin x$



Domain = ? ?   Range = ? ?

Trigonometric (5/16):

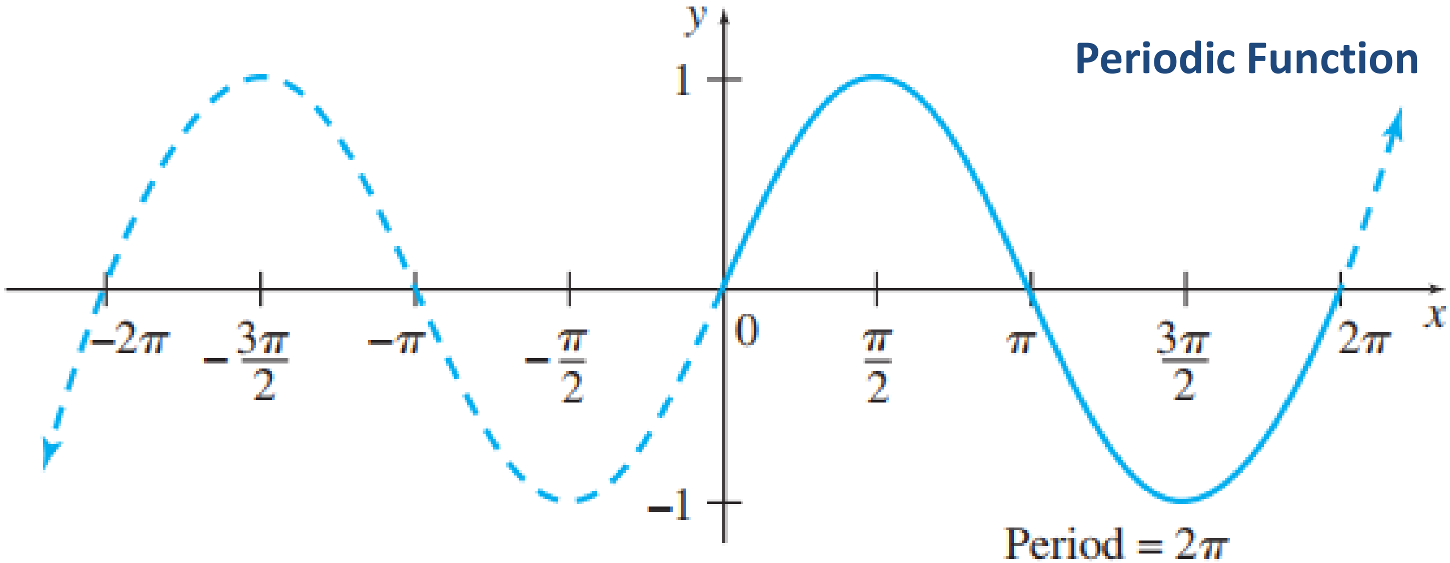
$f(x) = \sin x$



Domain = $\mathbb{R}$ Range = $[-1,1]$

Trigonometric (6/16):

$f(x) = \sin x$

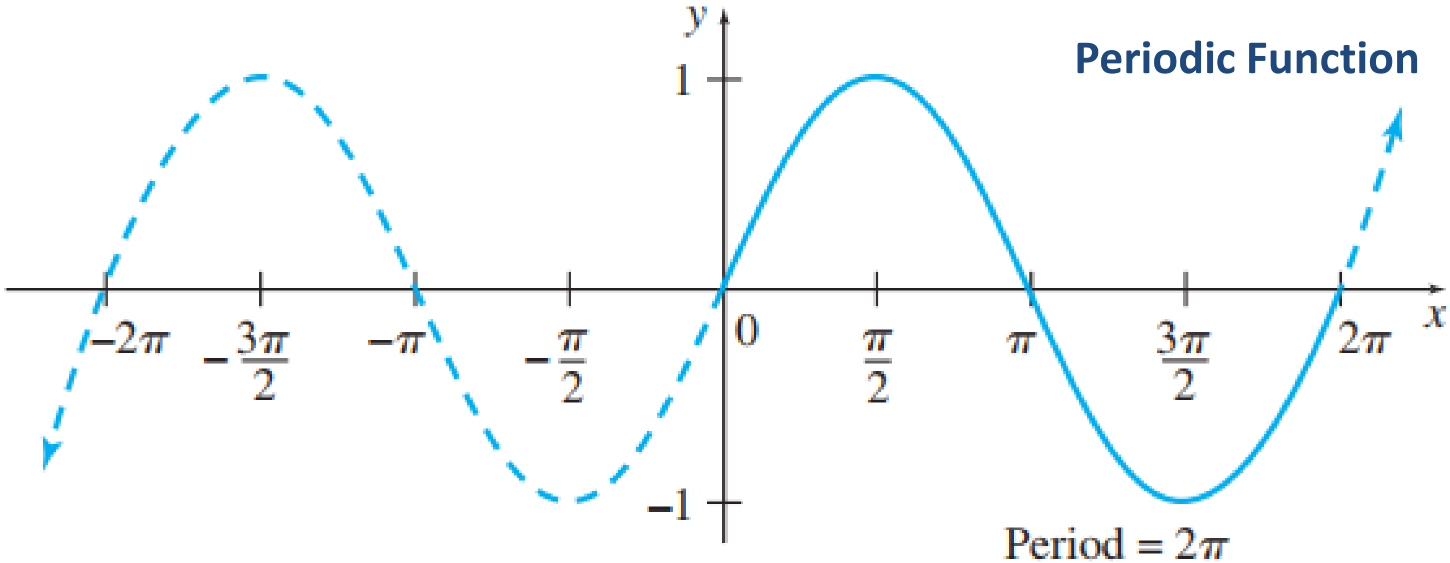


Odd or Even ??

Types of Fun. & Graph (16/18)

Trigonometric (6/16):

$f(x) = \sin x$



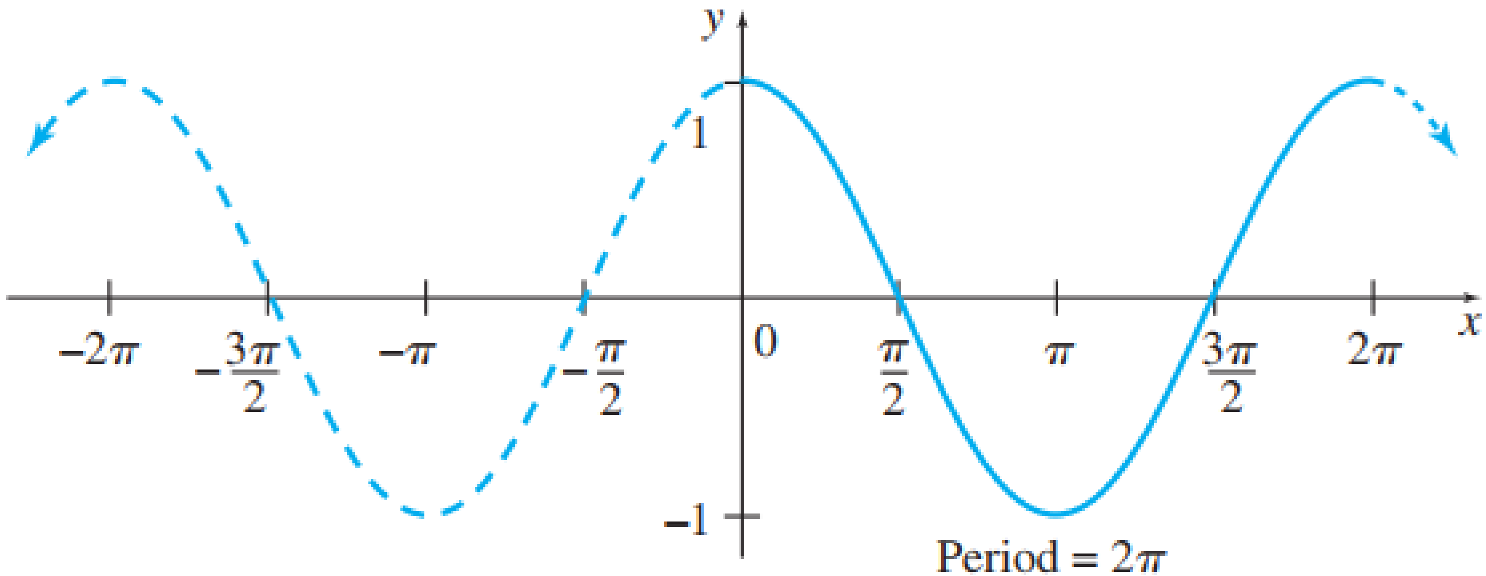
Odd

$f(-x) = -f(x)$

Trigonometric (7/16):

$f(x) = \cos x$

Periodic Function



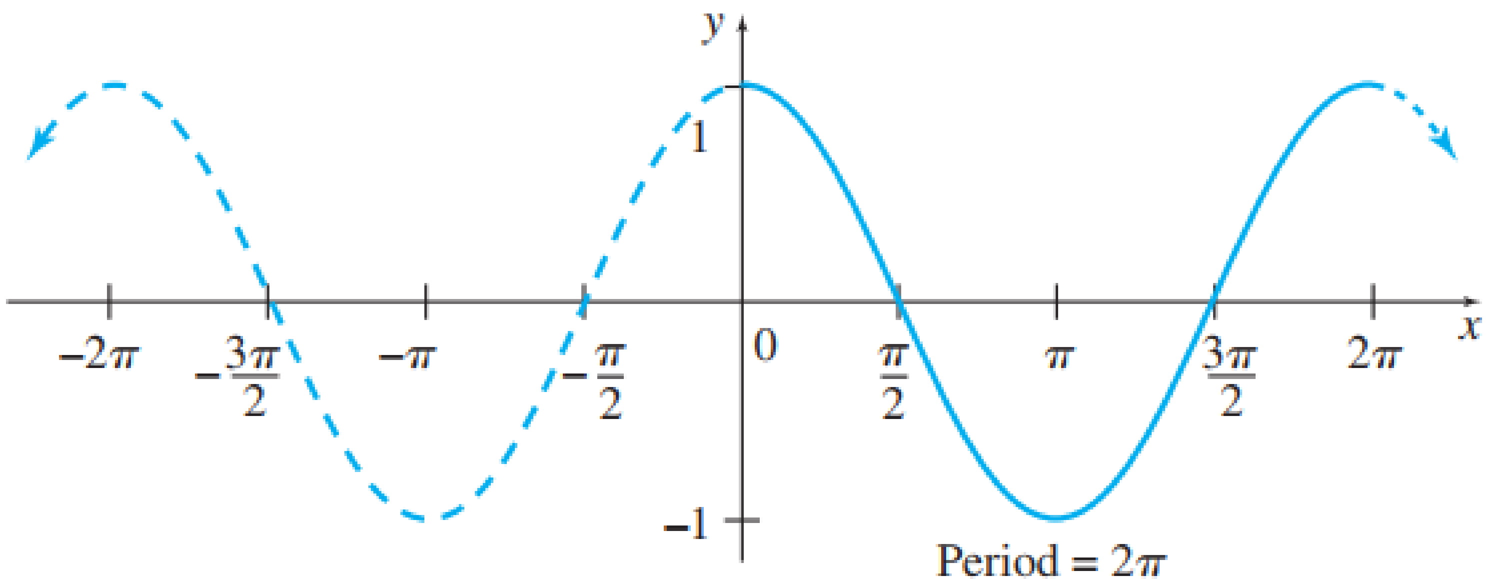
Types of Fun. & Graph (16/18)

Trigonometric (7/16):

Note that: $\cos(x \pm 2\pi) = \cos(x)$

$$f(x) = \cos x$$

Periodic Function

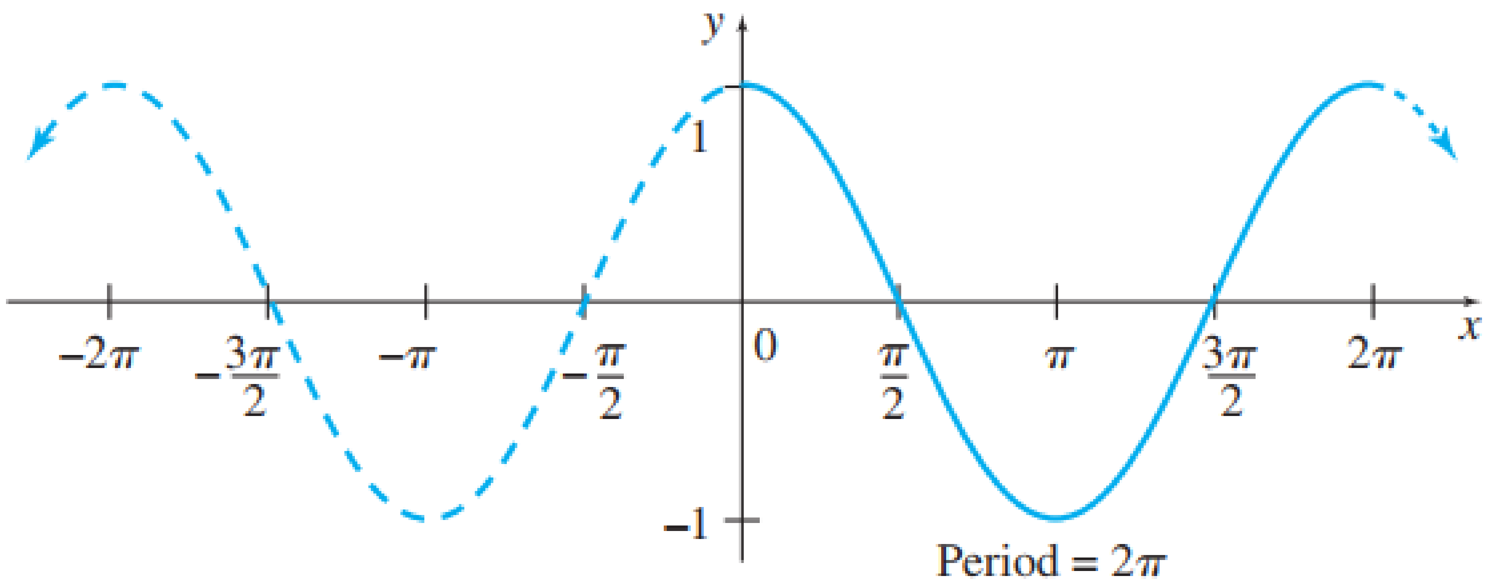


Types of Fun. & Graph (16/18)

Trigonometric (8/16):

$f(x) = \cos x$

Periodic Function



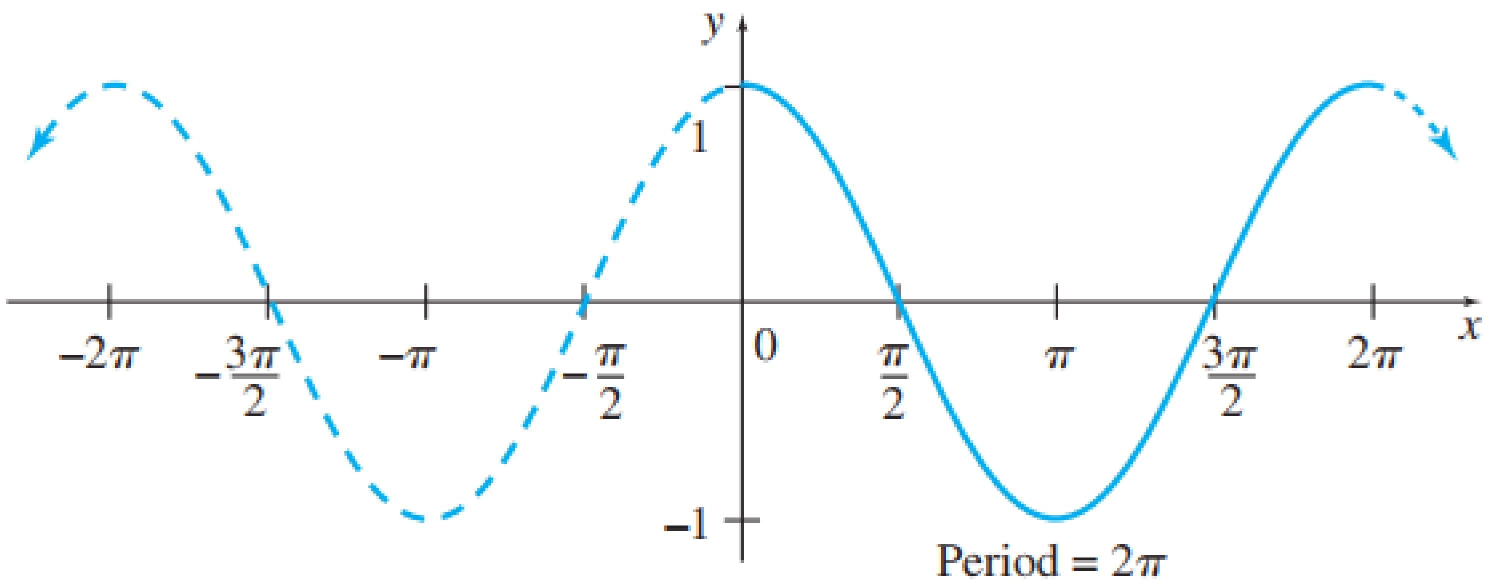
Domain = ? ?

Range = ? ?

Trigonometric (8/16):

$$f(x) = \cos x$$

Periodic Function

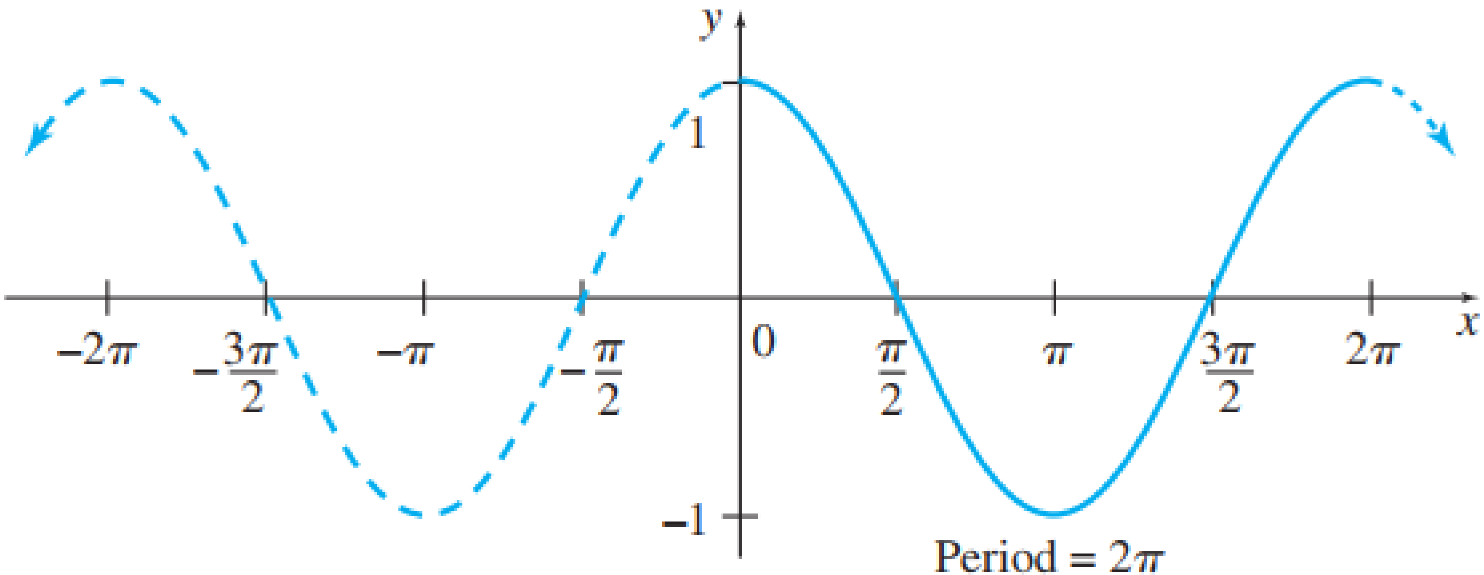


Domain = $\mathbb{R}$ Range = $[-1,1]$

Trigonometric (9/16):

$f(x) = \cos x$

Periodic Function

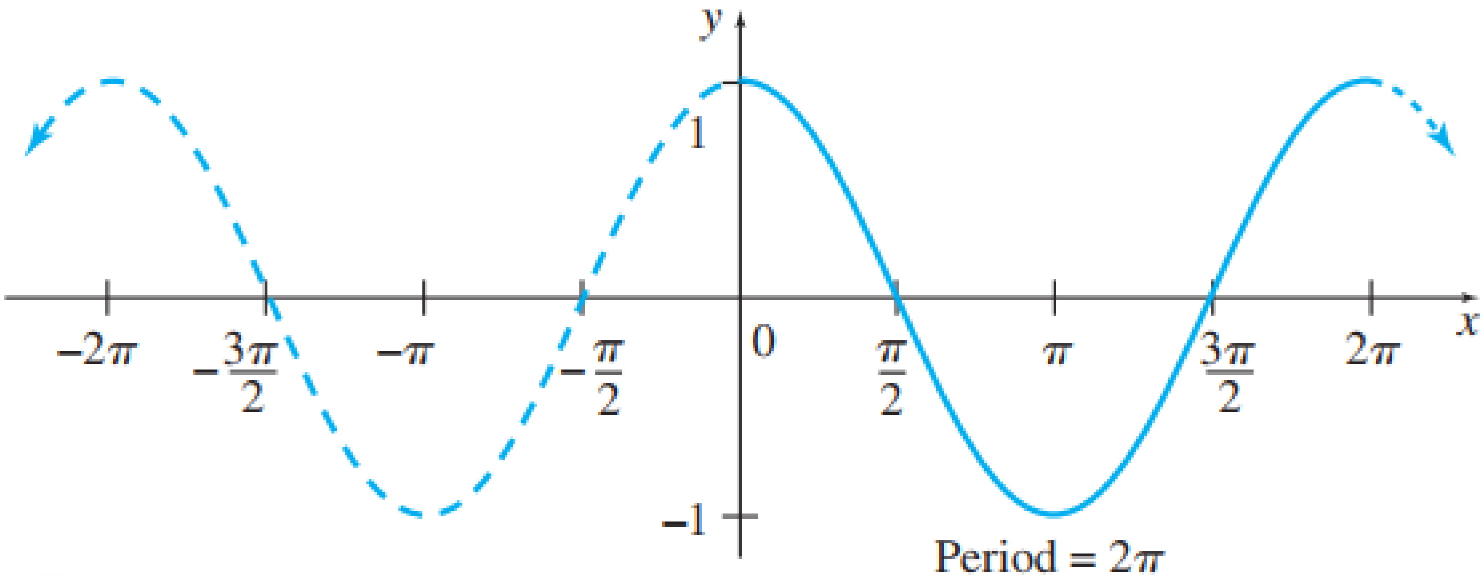


Odd or Even ??

Trigonometric (9/16):

$$f(x) = \cos x$$

Periodic Function



Even

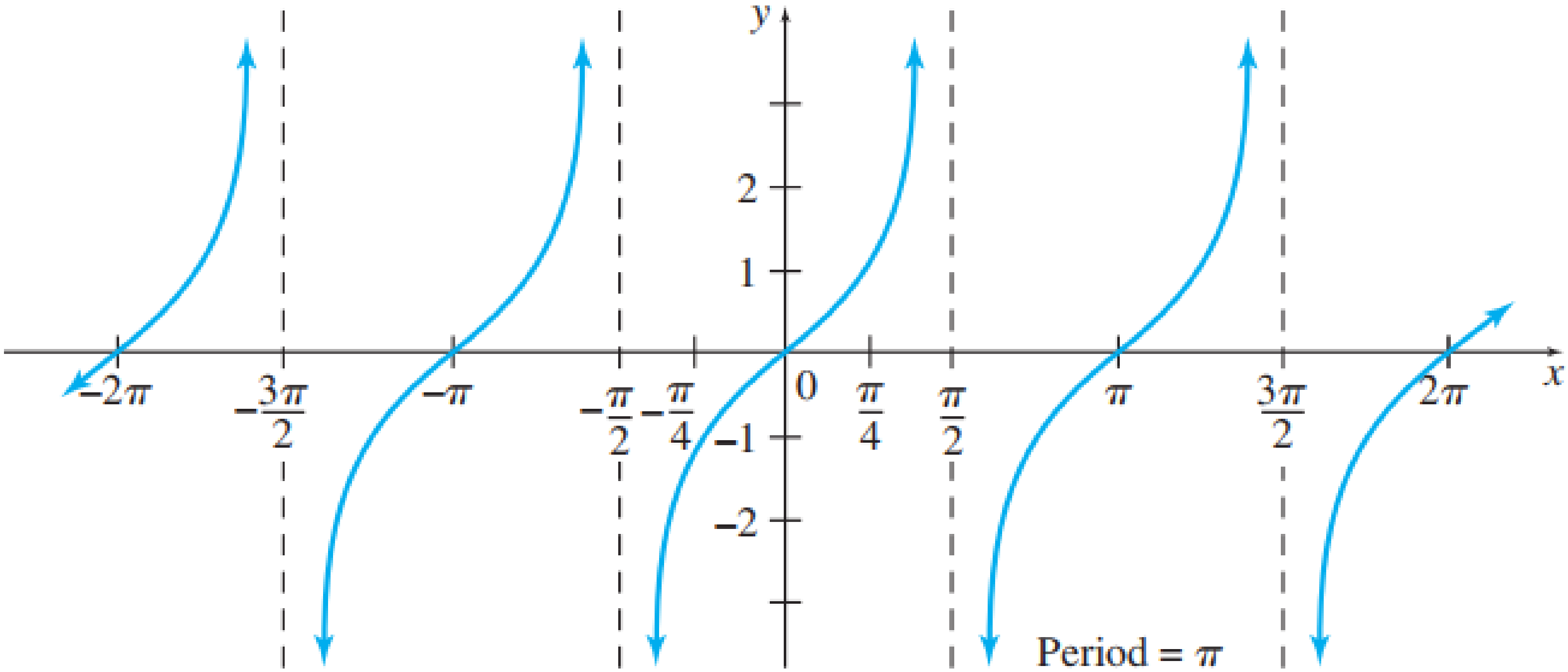
$$f(-x) = f(x)$$

Types of Fun. & Graph (16/18)

Trigonometric (10/16):

$f(x) = \tan x$

Periodic Function



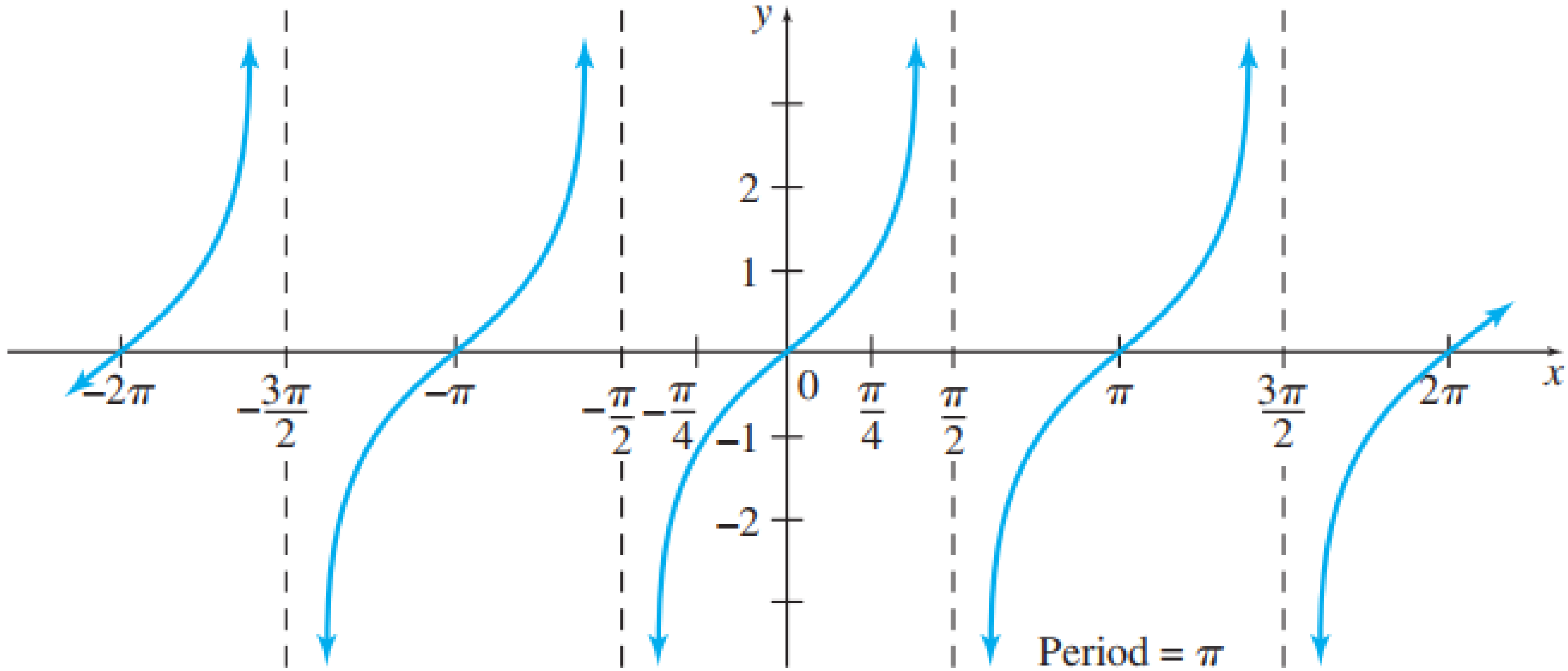
Types of Fun. & Graph (16/18)

Trigonometric (10/16):

Note that: $\tan(x \pm \pi) = \tan(x)$

$f(x) = \tan x$

Periodic Function

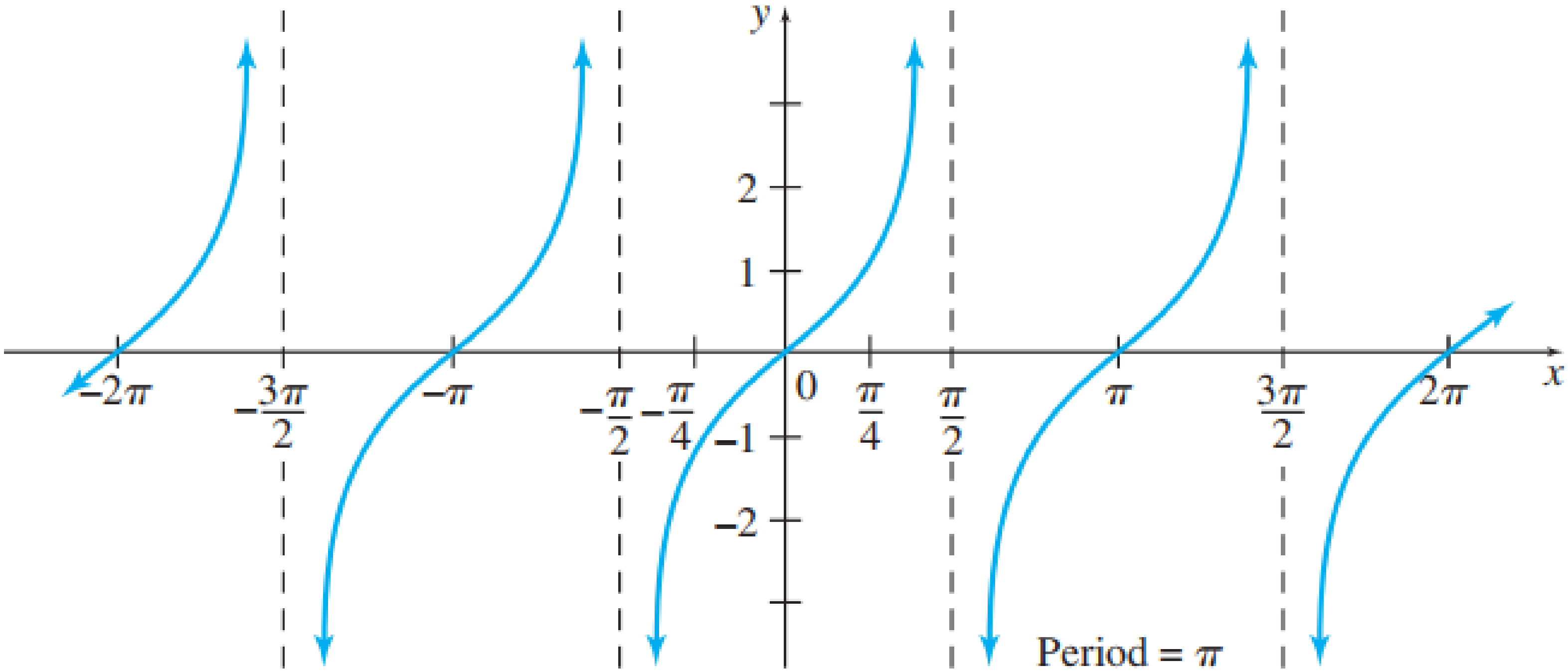


Types of Fun. & Graph (16/18)

Trigonometric (11/16):

$f(x) = \tan x$

Periodic Function



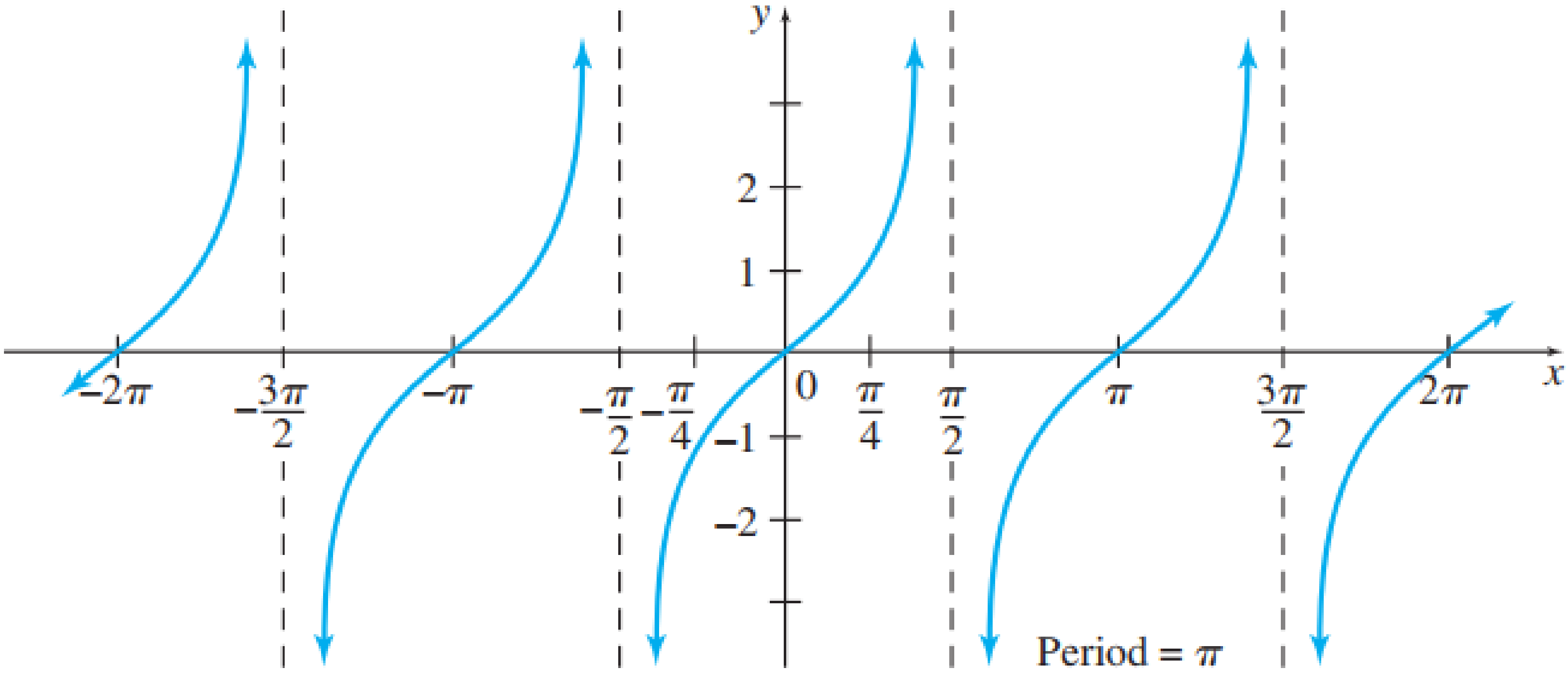
Domain = ? ?   Range = ? ?

Types of Fun. & Graph (16/18)

Trigonometric (11/16):

$f(x) = \tan x$

Periodic Function

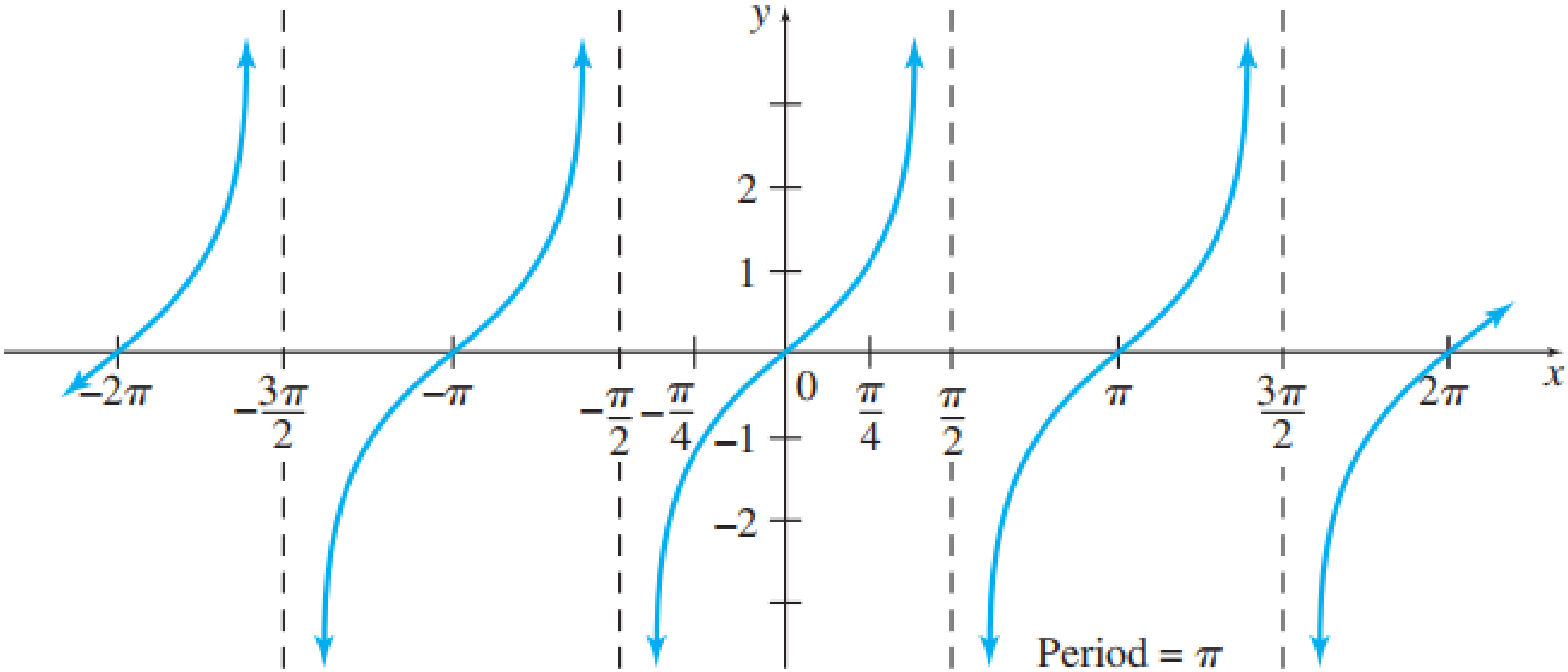


$$\text{Domain} = \mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots\} \qquad \text{Range} = \mathbb{R}$$

Trigonometric (12/16):

$f(x) = \tan x$

Periodic Function



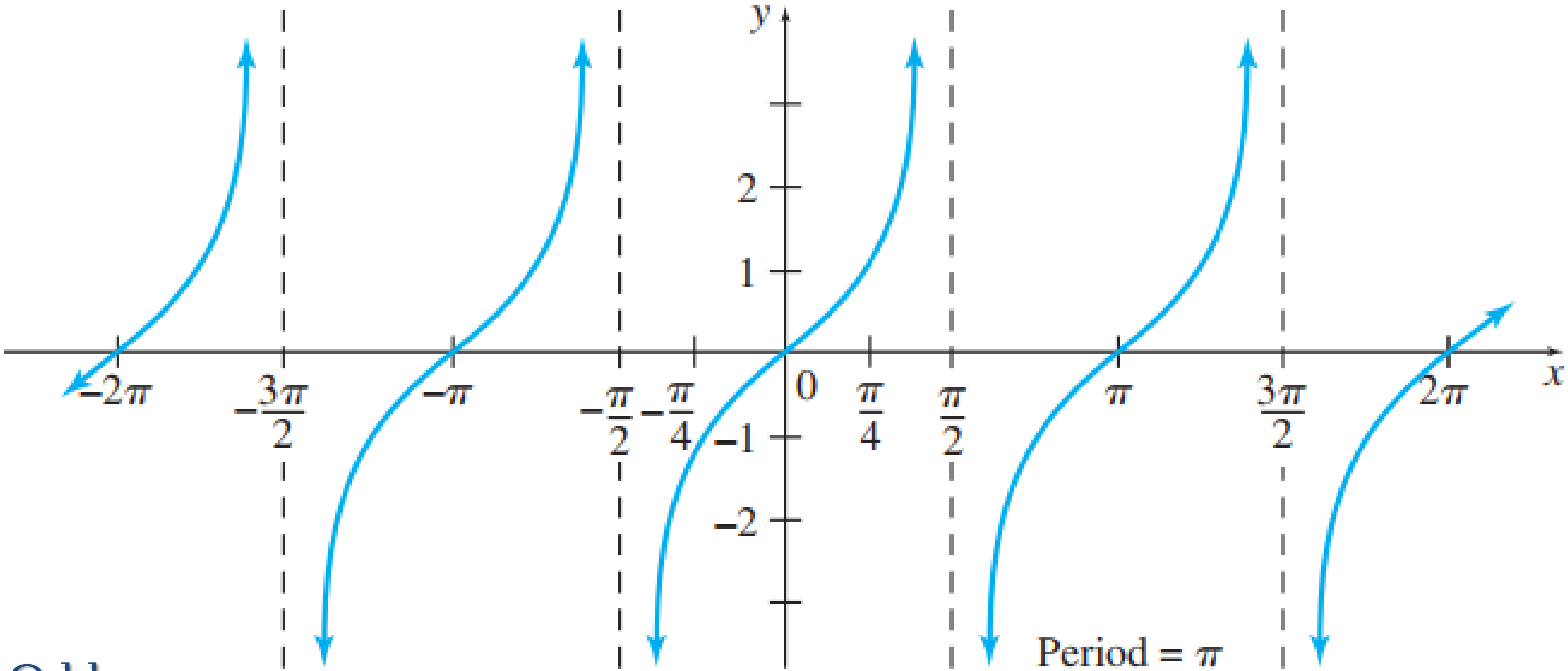
Odd or Even ??

Types of Fun. & Graph (16/18)

Trigonometric (12/16):

$f(x) = \tan x$

Periodic Function



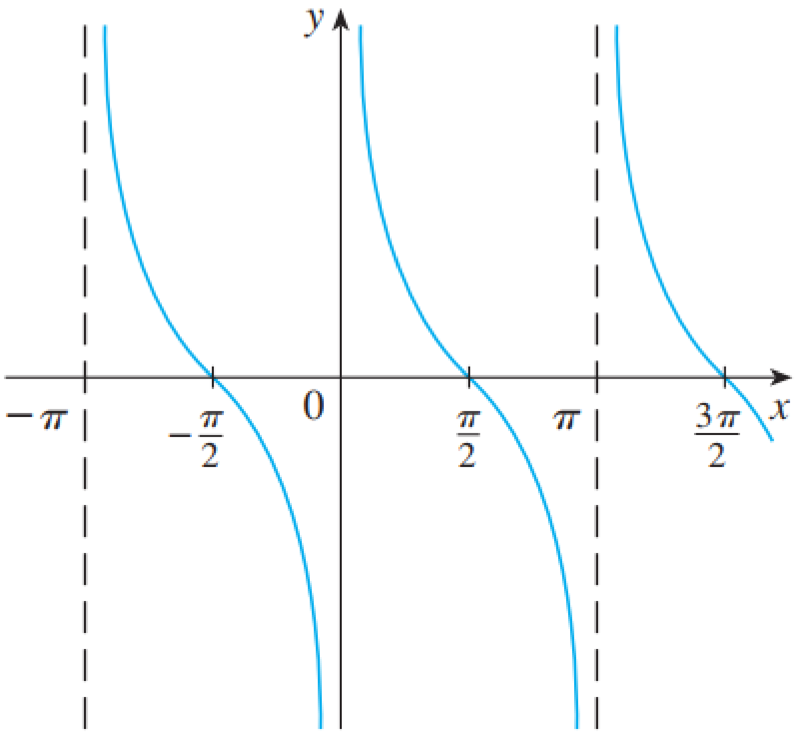
Odd

$f(-x) = -f(x)$

Types of Fun. & Graph (16/18)

Trigonometric (13/16):

$f(x) = \cot x$



Domain = ? ?

Range = ? ?

Odd or Even

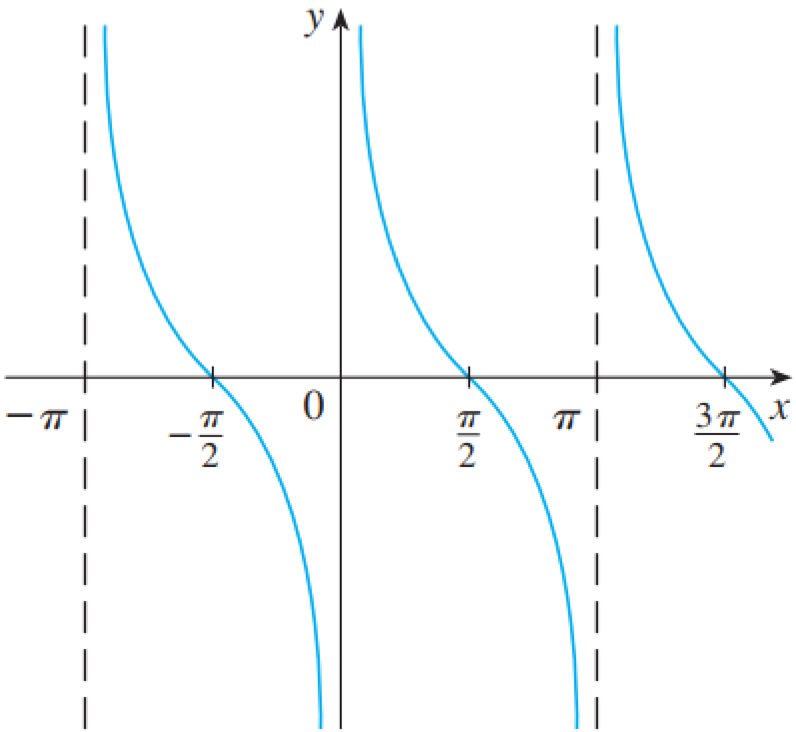
Periodic Function, Period = π

Types of Fun. & Graph (16/18)

Trigonometric (14/16):

Note that: $\cot(x \pm \pi) = \cot(x)$

$f(x) = \cot x$



Domain = $\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\}$

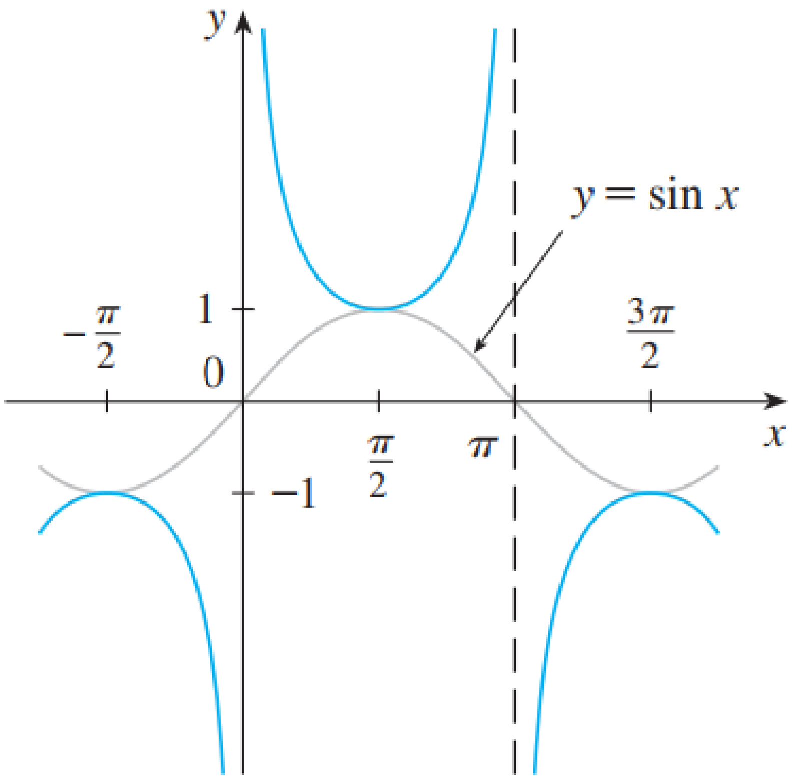
Range = $\mathbb{R}$

Odd $f(-x) = -f(x)$

Periodic Function, **Period = π**

Trigonometric (15/16):

$f(x) = \csc x$



Domain = ? ?

Range = ? ?

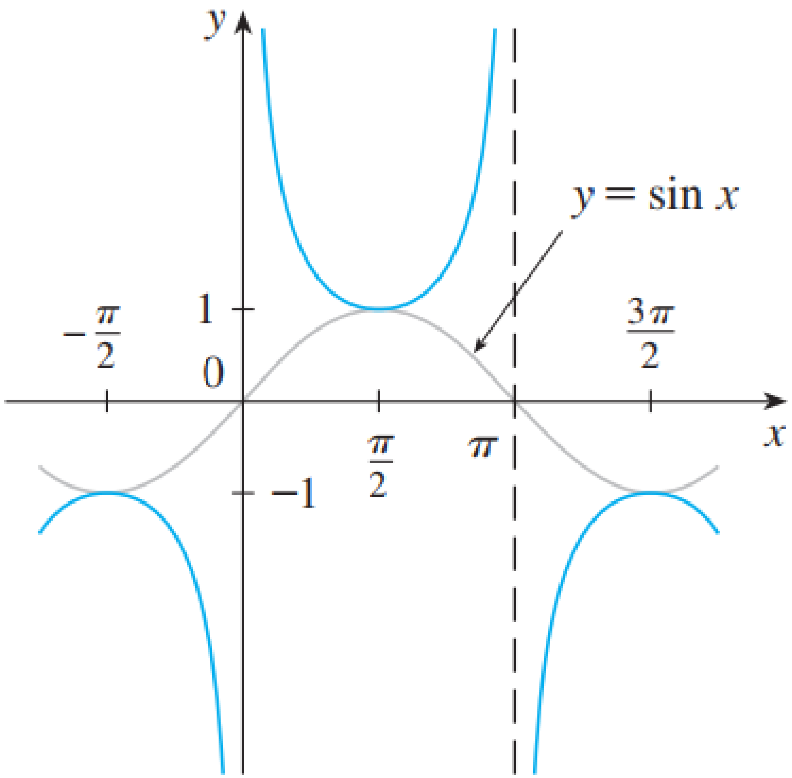
Odd or Even

Periodic Function, Period = 2π

Trigonometric (15/16):

Note that: $\csc(x \pm 2\pi) = \csc(x)$

$$f(x) = \csc x$$



Domain = $\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots\}$

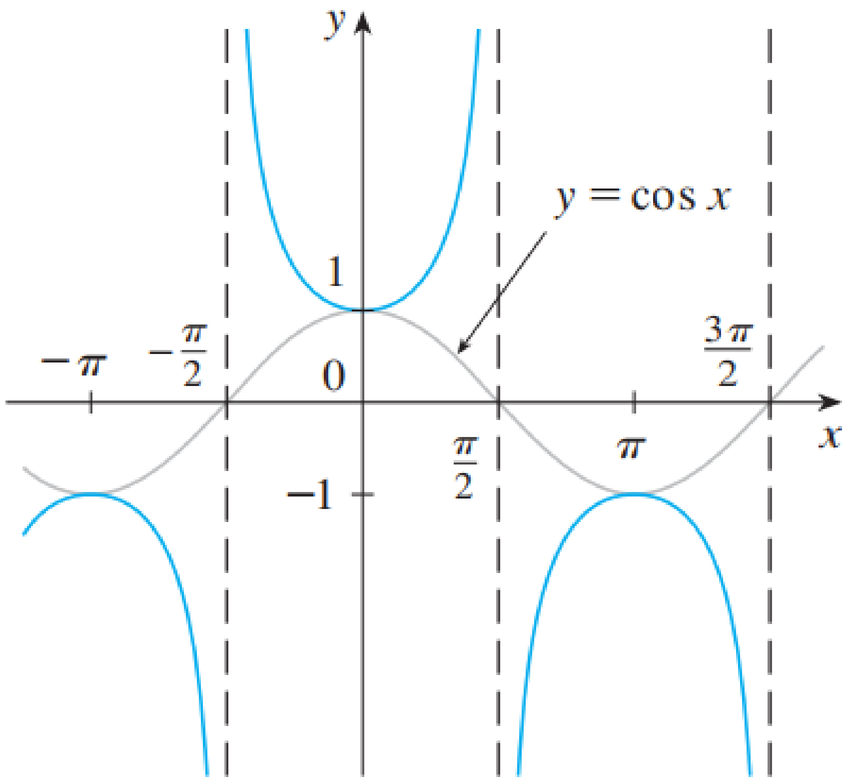
Range = $\mathbb{R} - (-1, 1)$

Odd $f(-x) = -f(x)$

Periodic Function, Period = 2π

Trigonometric (16/16):

$f(x) = \sec x$



Domain = ? ?

Range = ? ?

Odd or Even

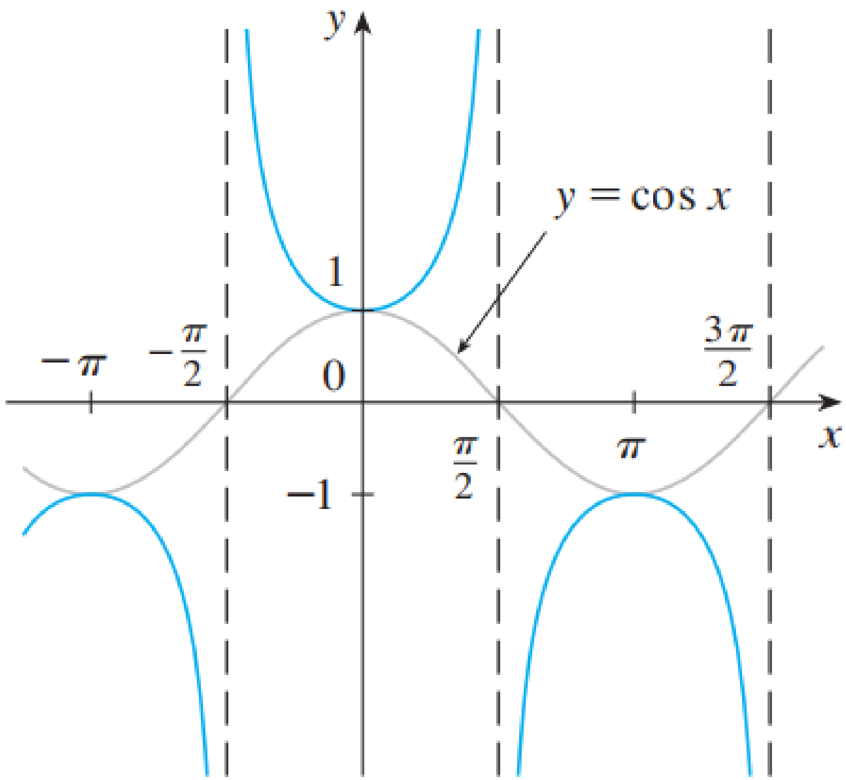
Periodic Function, Period = 2π

Types of Fun. & Graph (16/18)

Trigonometric (16/16):

Note that: $\sec(x \pm 2\pi) = \sec(x)$

$f(x) = \sec x$



Domain = $\mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots\}$

Range = $\mathbb{R} - (-1, 1)$

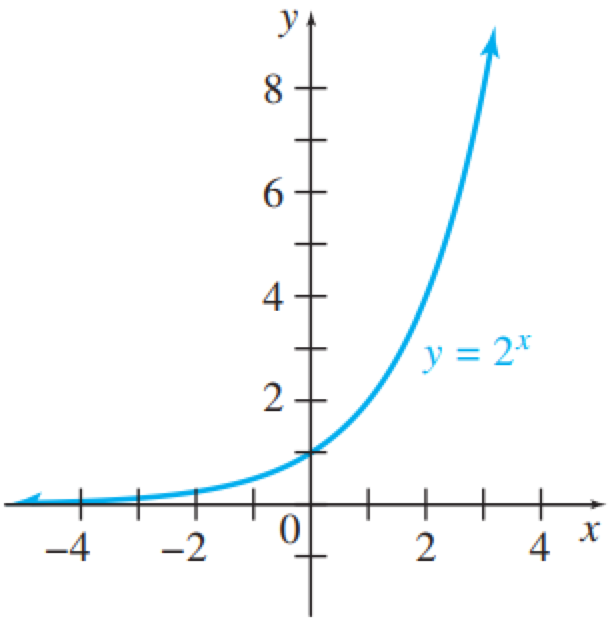
Even $f(-x) = f(x)$

Periodic Function, Period = 2π

Exponential (1/6):

The exponential functions are the functions of the form $f(x) = a^x$, where the base $a > 0$ and $a \neq 1$.

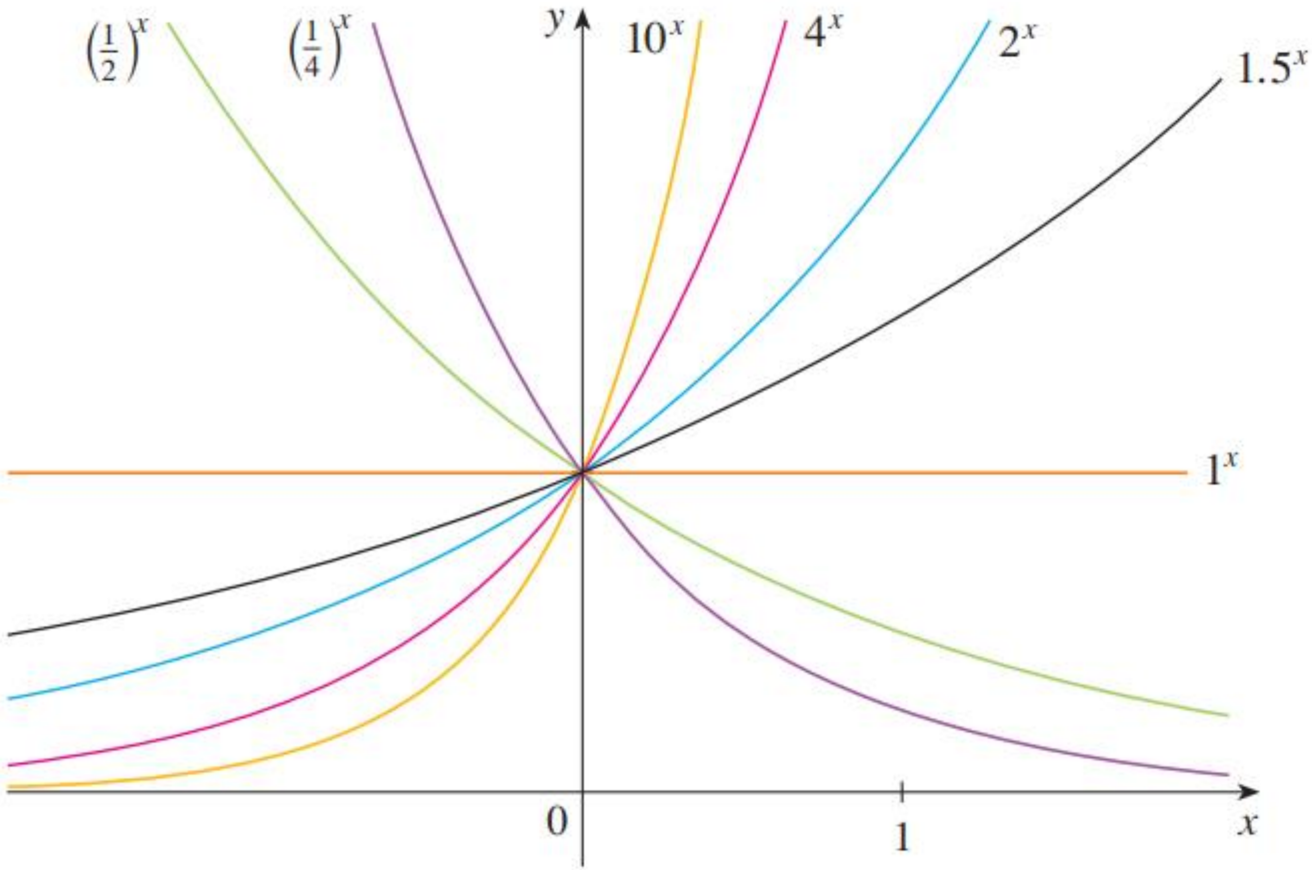
$f(x) = 2^x$



Types of Fun. & Graph (17/18)

Exponential (2/6):

$f(x) = a^x$



Exponential (3/6):

Laws of Exponents

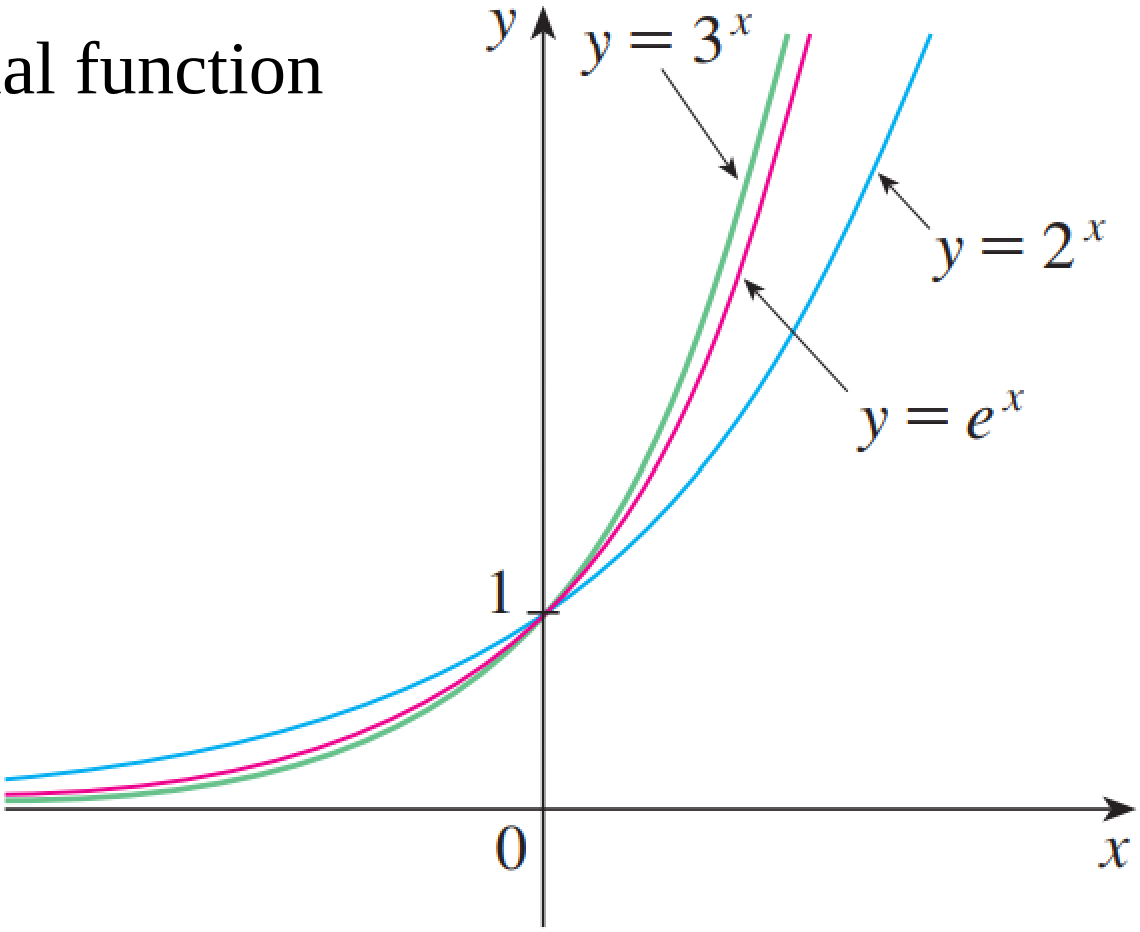
1. $b^r b^s = b^{r+s}$ Product rule.
2. $b^{-r} = \dfrac{1}{b^r}$ Changing sign of exponents.
3. $\dfrac{b^r}{b^s} = b^r \cdot b^{-s} = b^{r-s}$ Quotient rule.
4. $(b^r)^s = b^{rs}$ Power of a power.
5. $(ab)^r = a^r b^r$ Power of a product.
6. $\left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$ Power of a quotient.

Types of Fun. & Graph (17/18)

Exponential (4/6):

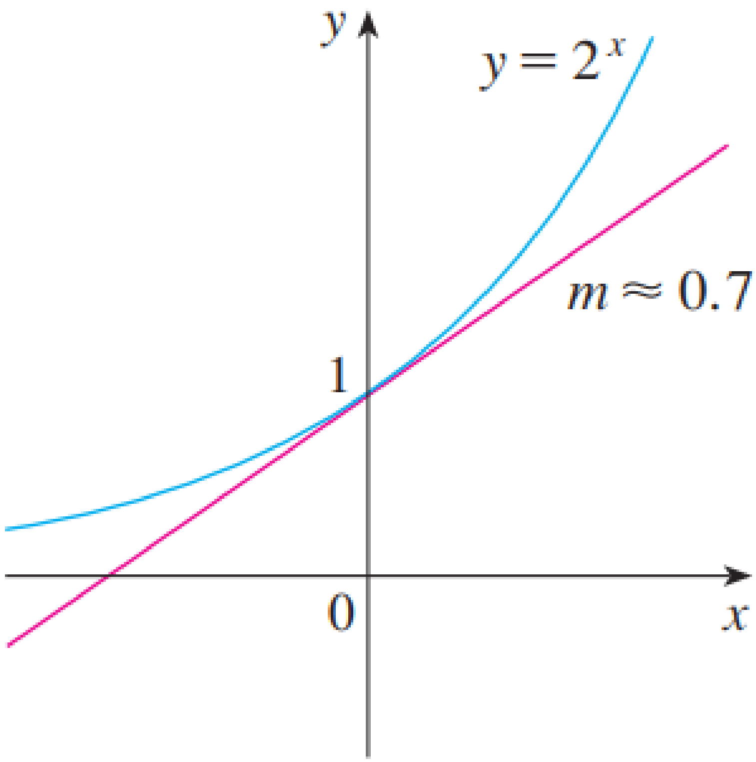
The natural exponential function

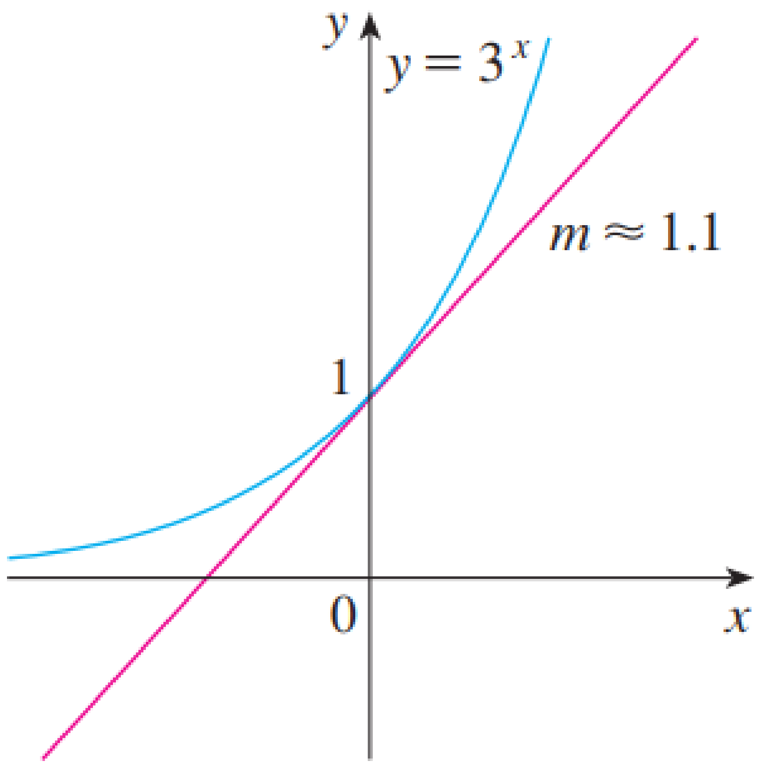
$f(x) = e^x$



Types of Fun. & Graph (17/18)

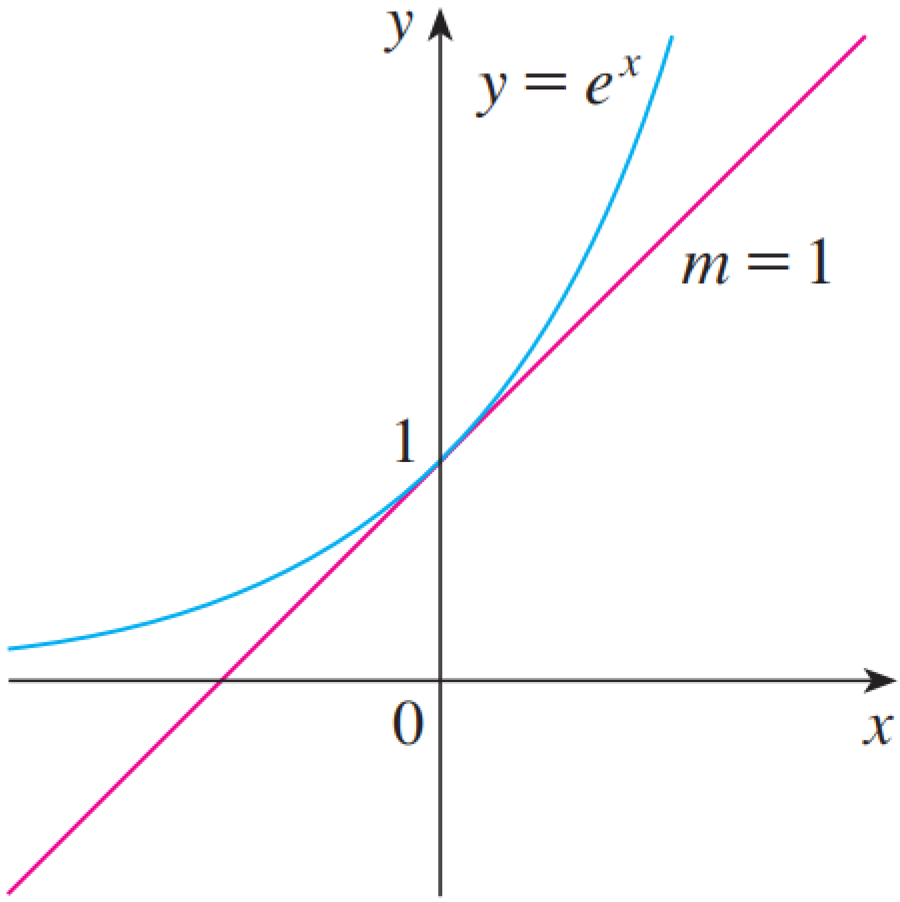
Exponential (5/6):





Types of Fun. & Graph (17/18)

Exponential (6/6):



Logarithmic (1/12):

$55 = (2)^x$, what is the value of x ??

Logarithmic (1/12):

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Logarithms are defined for just this purpose.

For $a > 0,\ a \neq 1,$ and $y > 0,$

$$\boldsymbol{x = \log_a y} \quad \text{means} \quad \boldsymbol{y = a^x}$$

Logarithmic (1/12):

$55 = (2)^x$, what is the value of x ??

Logarithms are defined for just this purpose.

For $a > 0,\ a \neq 1,$ and $y > 0,$

$$\boldsymbol{x = \log_a y} \qquad \text{means} \qquad \boldsymbol{y = a^x}$$

$55 = (2)^x \;\rightarrow\; x = \log_2 55 \approx 5.78135971352466$

Logarithmic (2/12):

Logarithm's function $f(x) = \log_a x$.

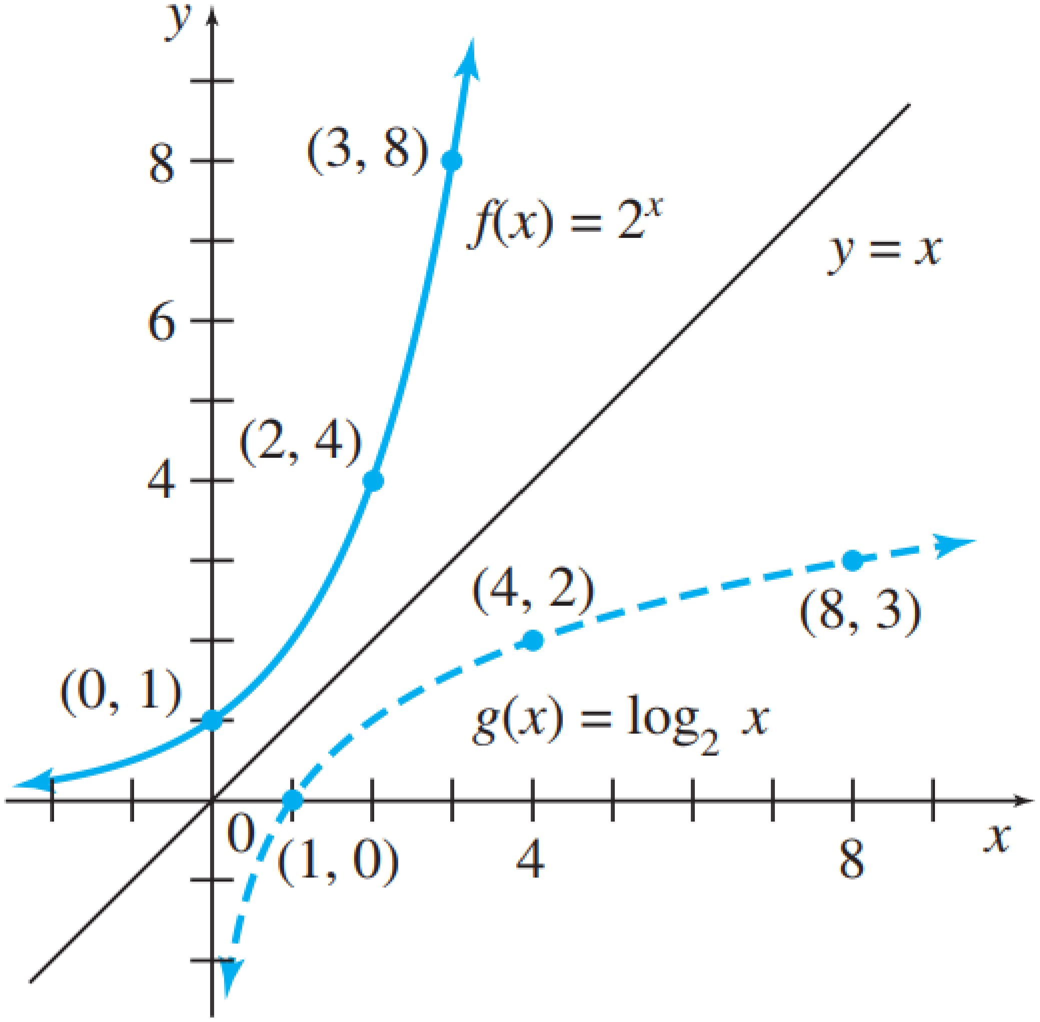
For $a > 0$, $a \neq 1$, and $x > 0$, the **logarithmic function** of base a is defined by

$$\boldsymbol{f(x) = \log_a x}$$

Types of Fun. & Graph (18/18)

Logarithmic (3/12):

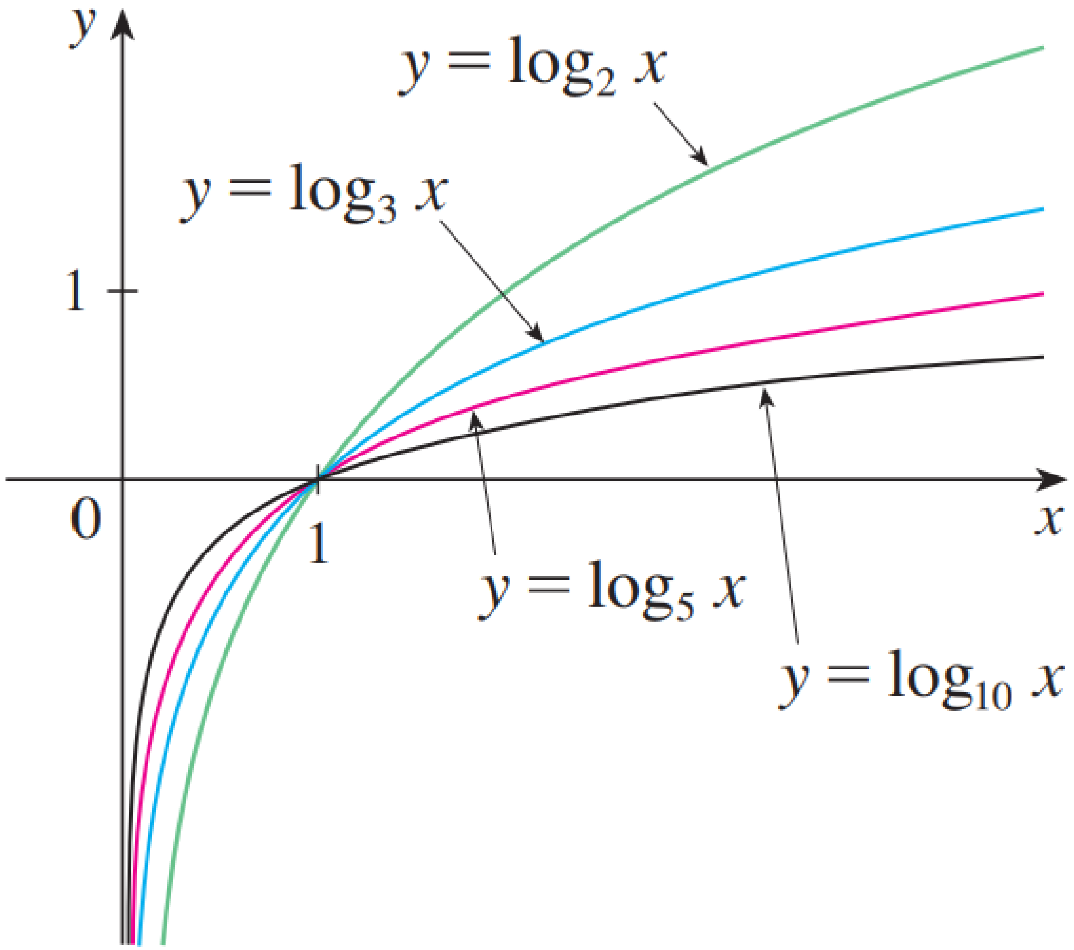
$$g(x) = \log_2 x.$$



Types of Fun. & Graph (18/18)

Logarithmic (4/12):

$y = \log_a x.$



Types of Fun. & Graph (18/18)

Logarithmic (5/12):

Properties of Logarithms: Let x and y be any positive real numbers and r be any real number. Let a be a positive real number, $a \neq 1$. Then:

(a) $\log_a xy = \log_a x + \log_a y$

(b) $\log_a \frac{x}{y} = \log_a x - \log_a y$

(c) $\log_a x^r = r \log_a x$

(d) $\log_a a = 1$

(e) $\log_a 1 = 0$

(f) $\log_a a^r = r.$

Logarithmic (6/12):

Base 10 logarithms were called common logarithms.

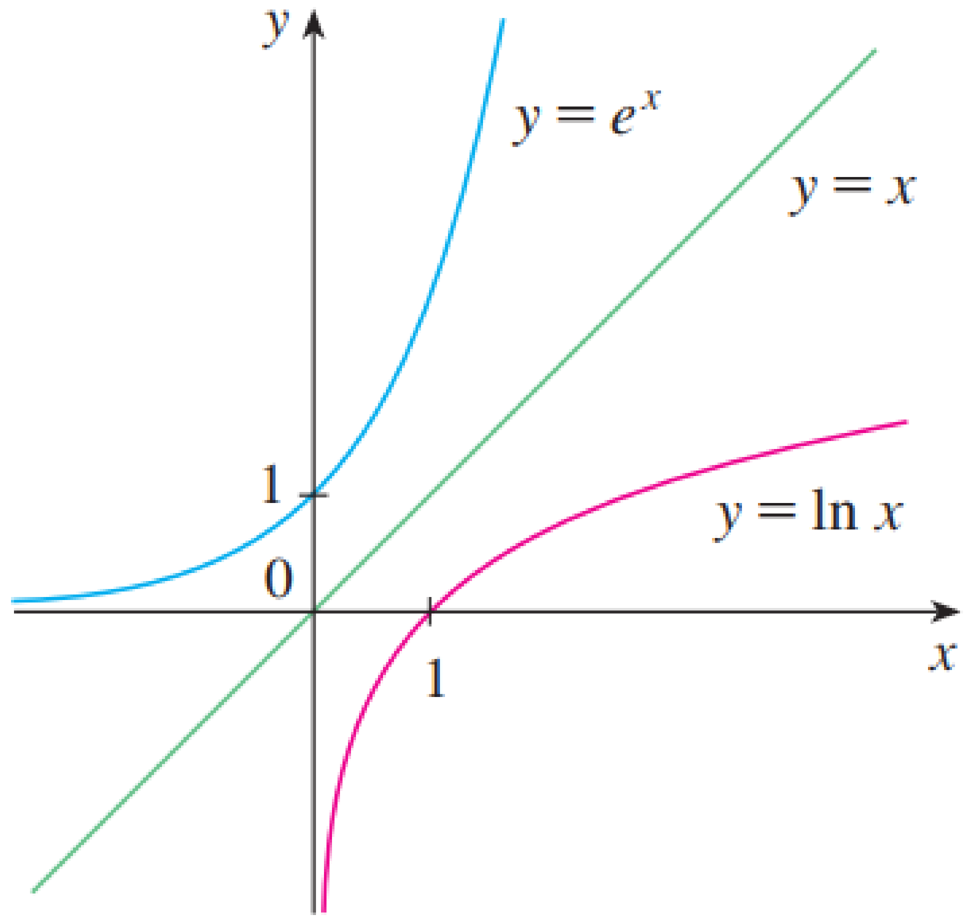
$\log_{10} x$ is abbreviated $\log x$

Base e logarithms were called natural logarithms.

$\log_e x$ is abbreviated $\ln x$

Logarithmic (7/12):

$f(x) = \ln x$



Logarithmic (8/12):

Base e logarithms were called natural logarithms.

$\mathbf{log}_{e}\,\boldsymbol{x}$ is abbreviated $\mathbf{ln}\,\boldsymbol{x}$

$$y = \ln x \quad \rightarrow \quad e^{y} = x$$

$$\log_{e} e = 1 \quad \rightarrow \quad \ln e = 1$$

$$e^{\ln x} = x$$

Logarithmic (9/12):

Find the value of x if $\mathbf{3^x = 5}$

$$\ln 3^x = \ln 5$$

$$x \ln 3 = \ln 5$$

$$x = \frac{\ln 5}{\ln 3} \approx 1.464973520718$$

Logarithmic (10/12):

Find the value of x if $\ln\left(\frac{x}{2}\right) = 5$

Logarithmic (11/12):

Find the value of x if $\mathbf{ln}\left(\frac{x}{2}\right) = \mathbf{5}$

$\ln x - \ln 2 = 5$

$\ln x = 5 + \ln 2$

$\ln x = 5 + \ln 2 \approx 5.693147181$

$e^{\ln x} = e^{5.693147181}$

$x = e^{5.693147181} \approx 296.82631834$

Logarithmic (12/12):

Change of Base Formula

If x is any positive number and if a and b are positive real numbers, $a \neq 1$, $b \neq 1$, then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Find the value $\mathbf{\log_2 37}$?

Logarithmic (12/12):

Change of Base Formula

If x is any positive number and if a and b are positive real numbers, $a \neq 1$, $b \neq 1$, then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Find the value $\mathbf{\log_2 37}$?

$$= \frac{\ln 37}{\ln 2} = \frac{\log 37}{\log 2} \approx 5.20945336563$$

Transformations of Funs. (1/2)

Vertical and Horizontal Shifts (1/4):

- Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

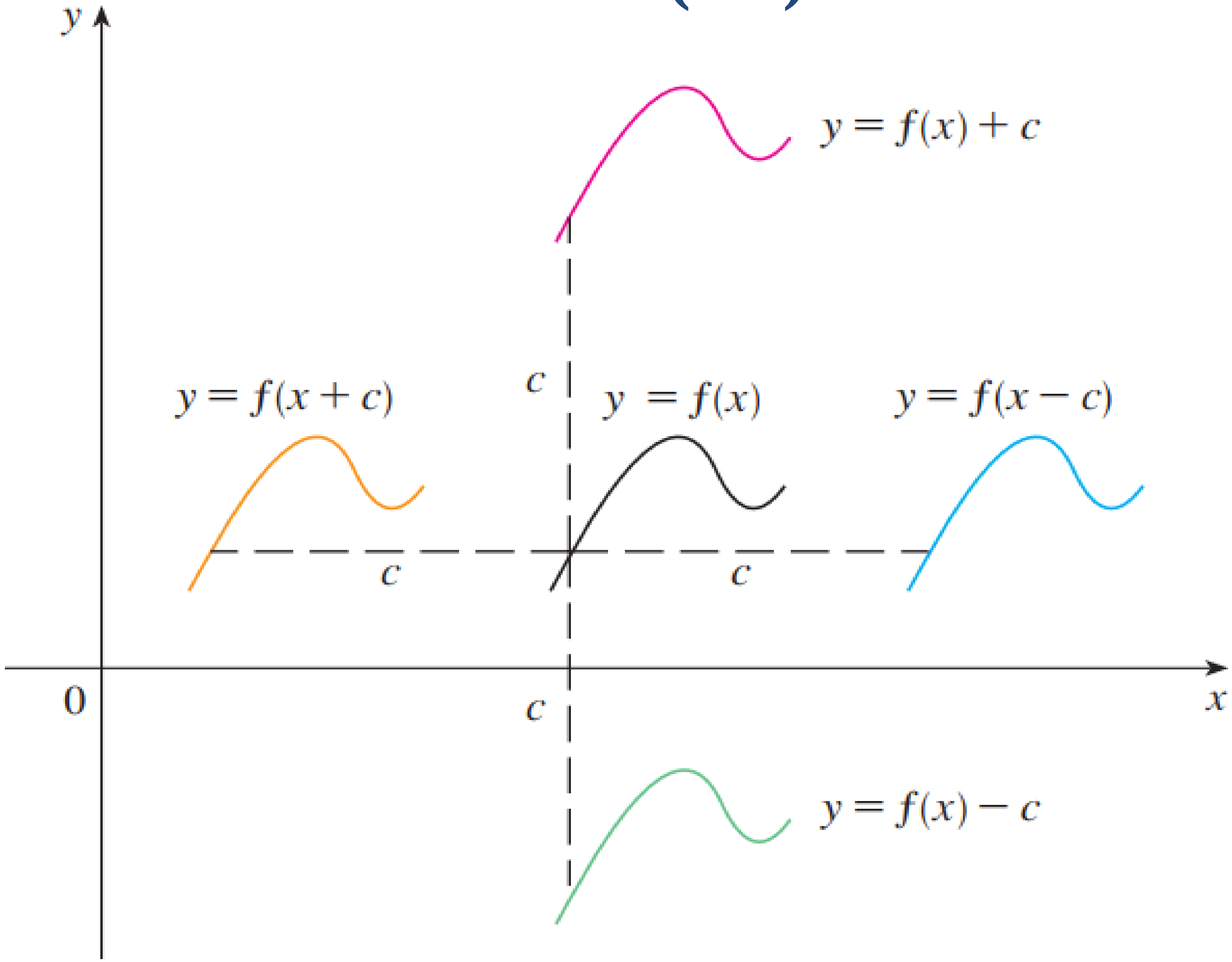
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

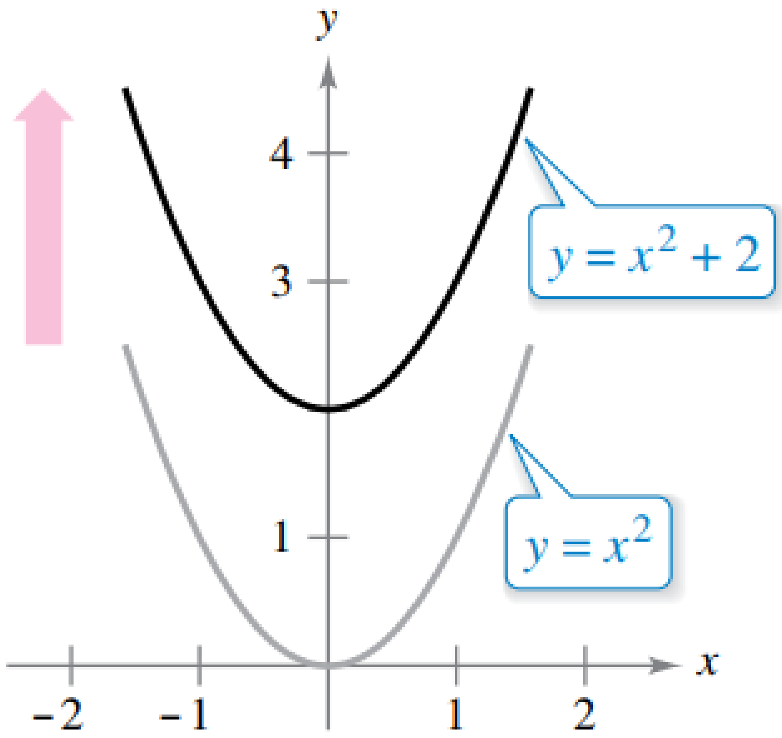
Transformations of Funs. (1/2)

Vertical and Horizontal Shifts (2/4):



Vertical and Horizontal Shifts (3/4):

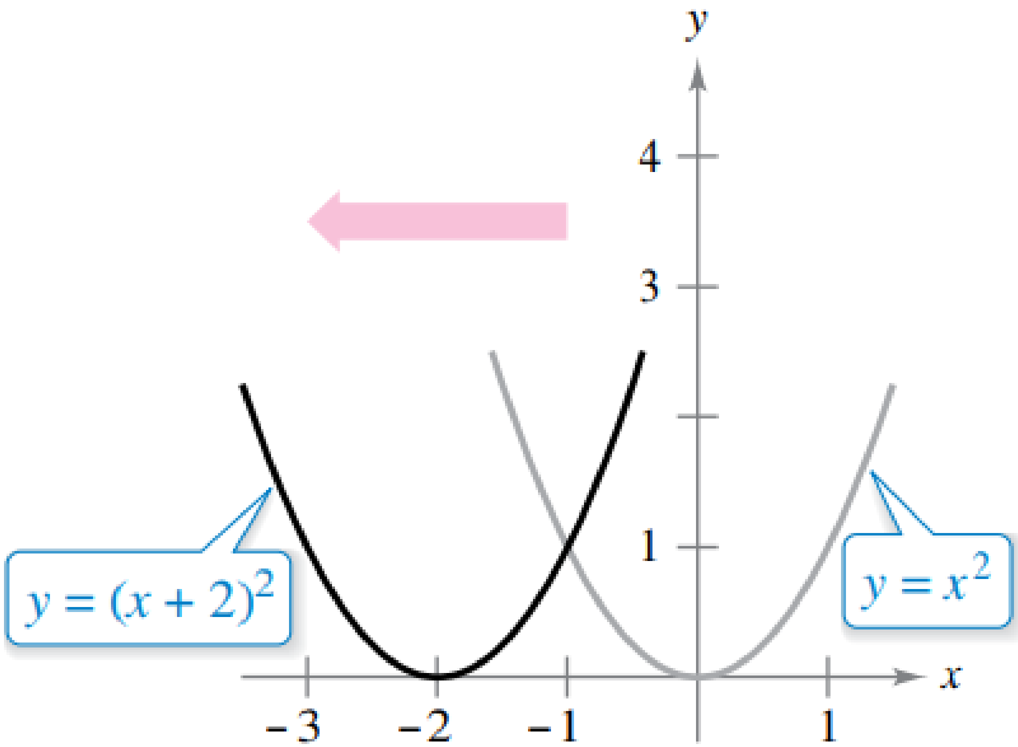
- $y = x^2 + 2$



Transformations of Funs. (1/2)

Vertical and Horizontal Shifts (4/4):

- $y = (x+2)^2$



Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (1/6):

- Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c

$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c

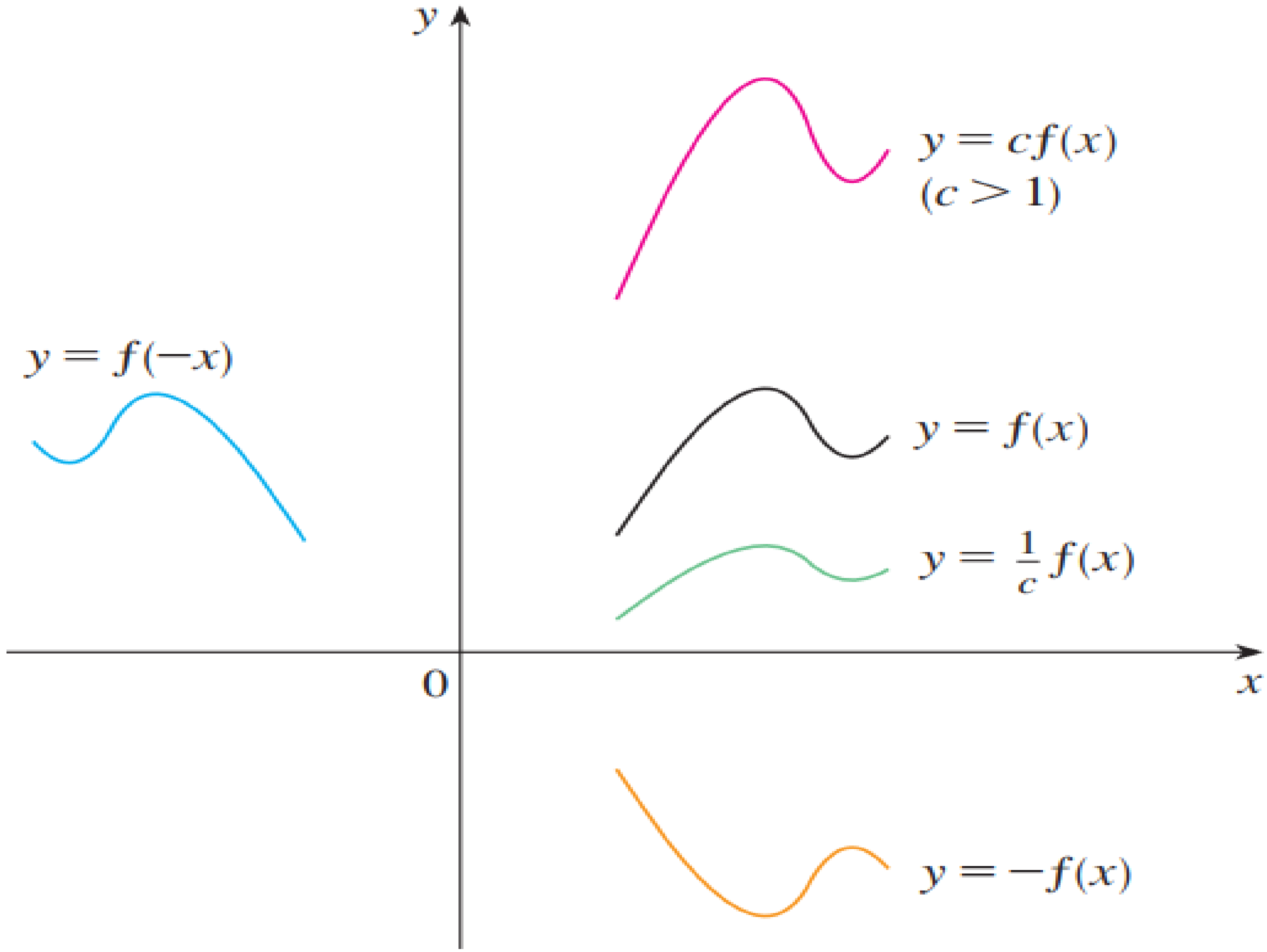
$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c

$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

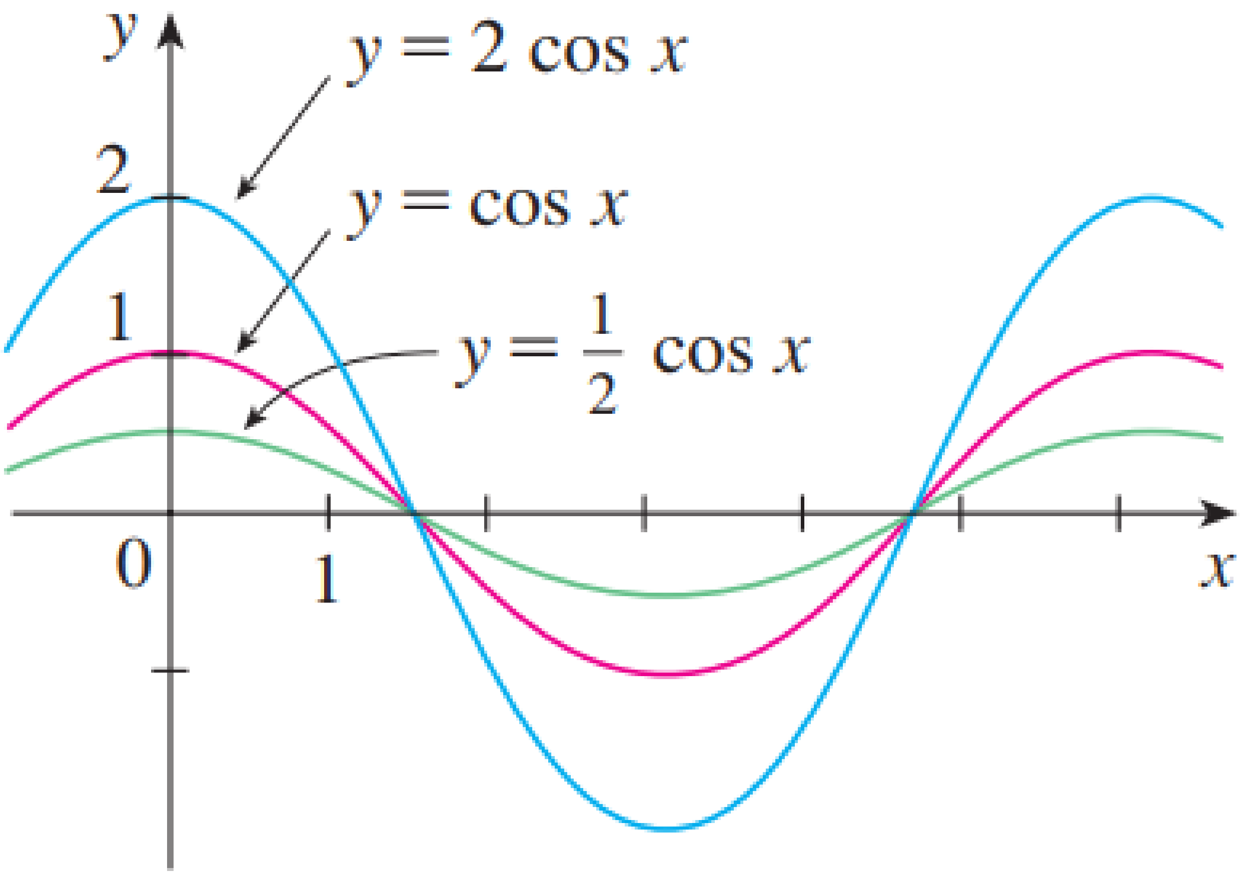
Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (2/6):



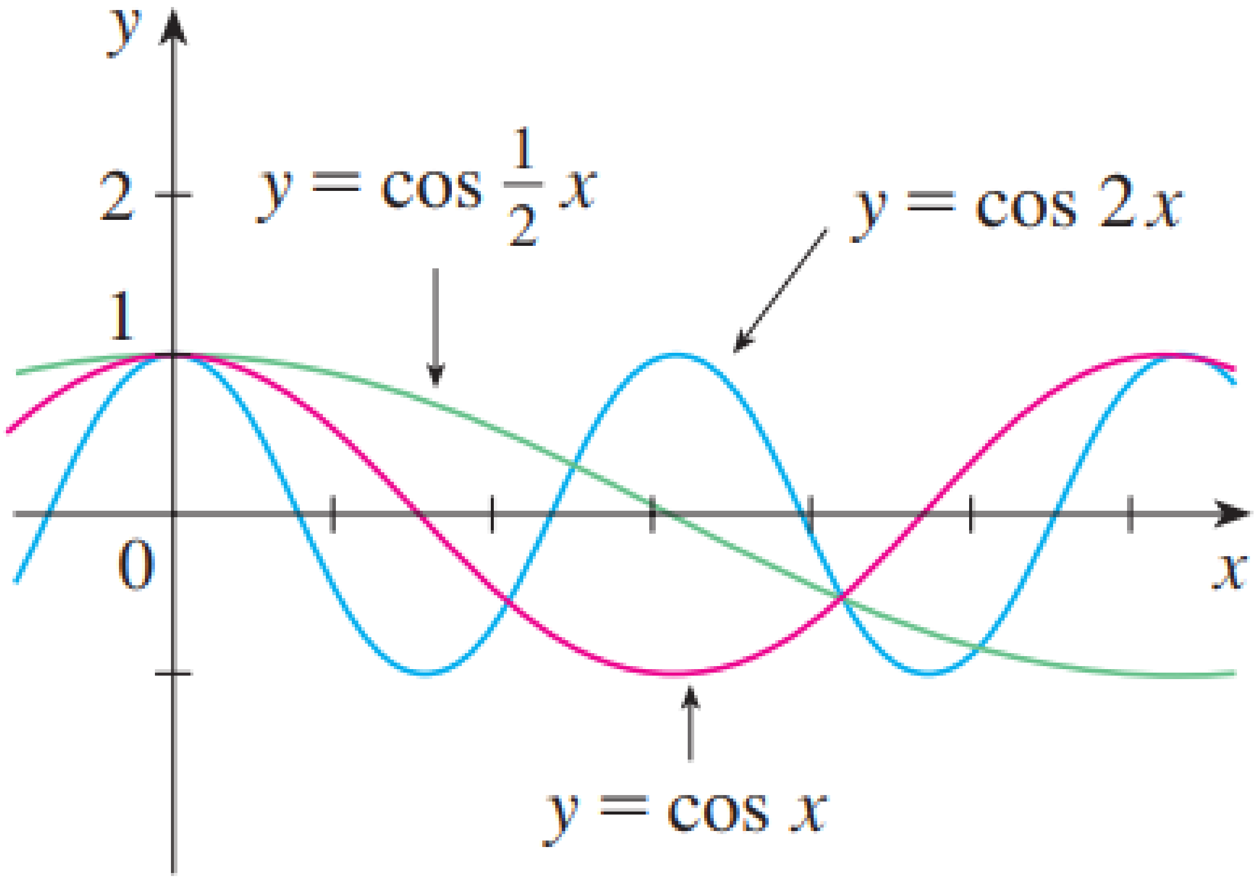
Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (3/6):

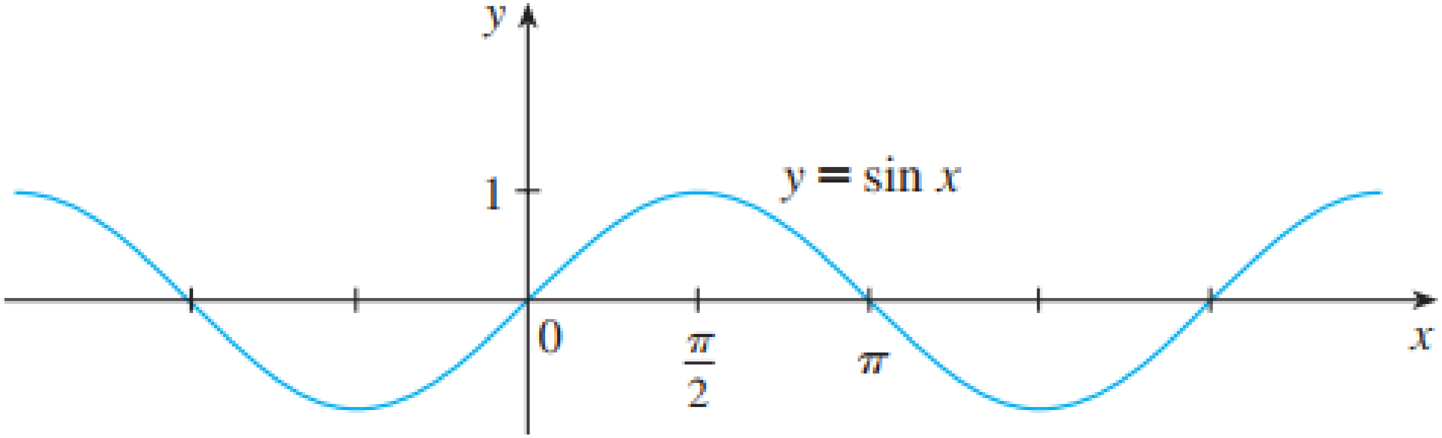


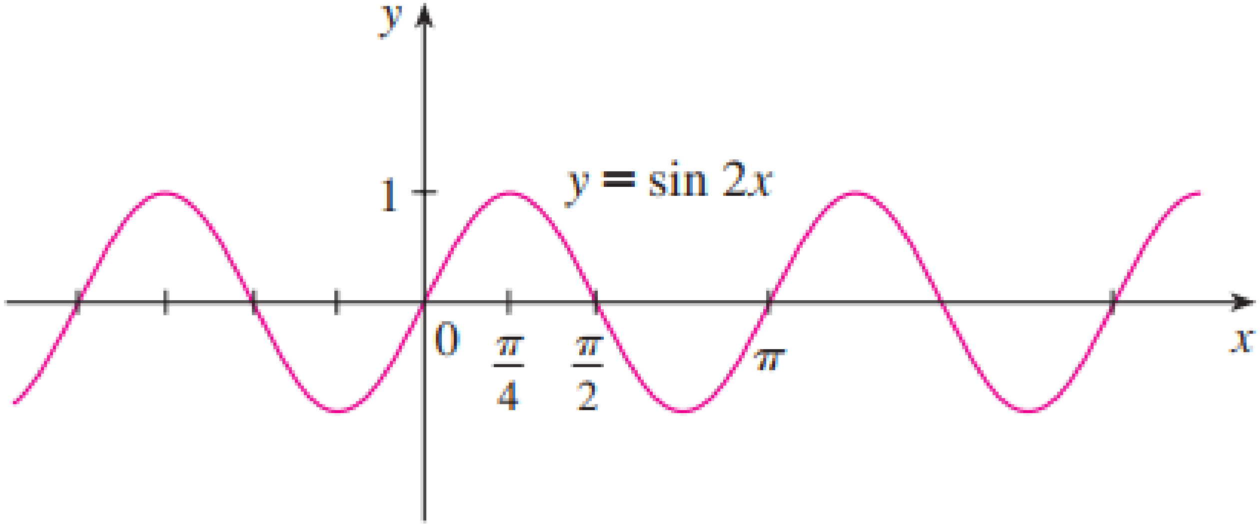
Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (3/6):



Vertical and Horizontal Stretching and Reflecting (4/6):

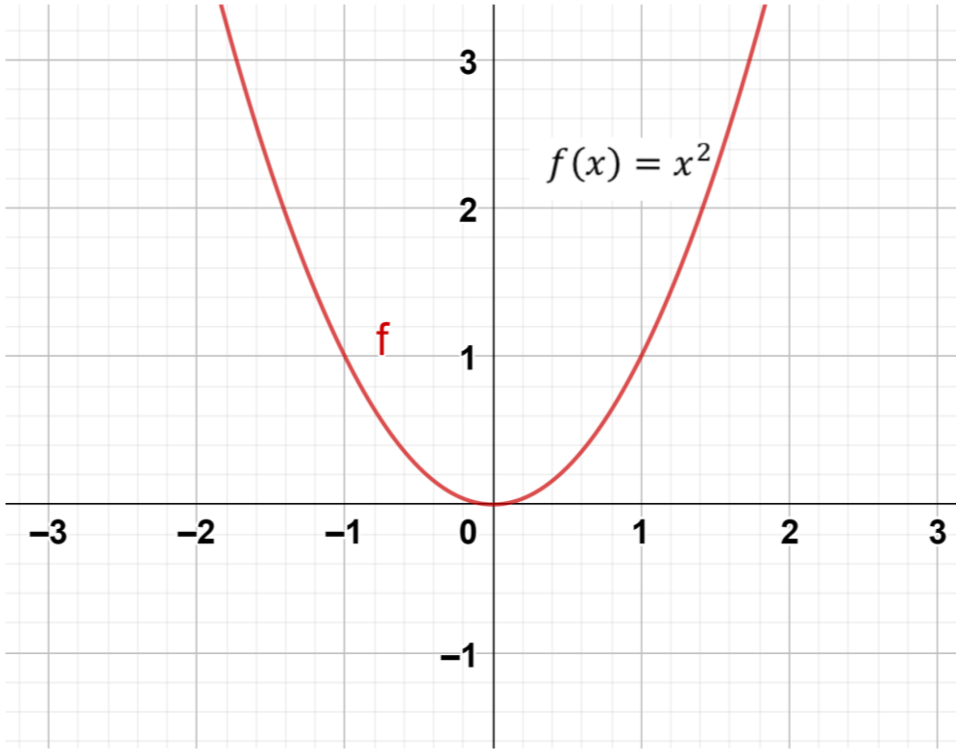




Vertical and Horizontal Stretching and Reflecting (5/6):

Starting with $f(x) = x^2$ to graph $g(x) = (x-1)^2 + 2$

$$f(x) = x^2$$

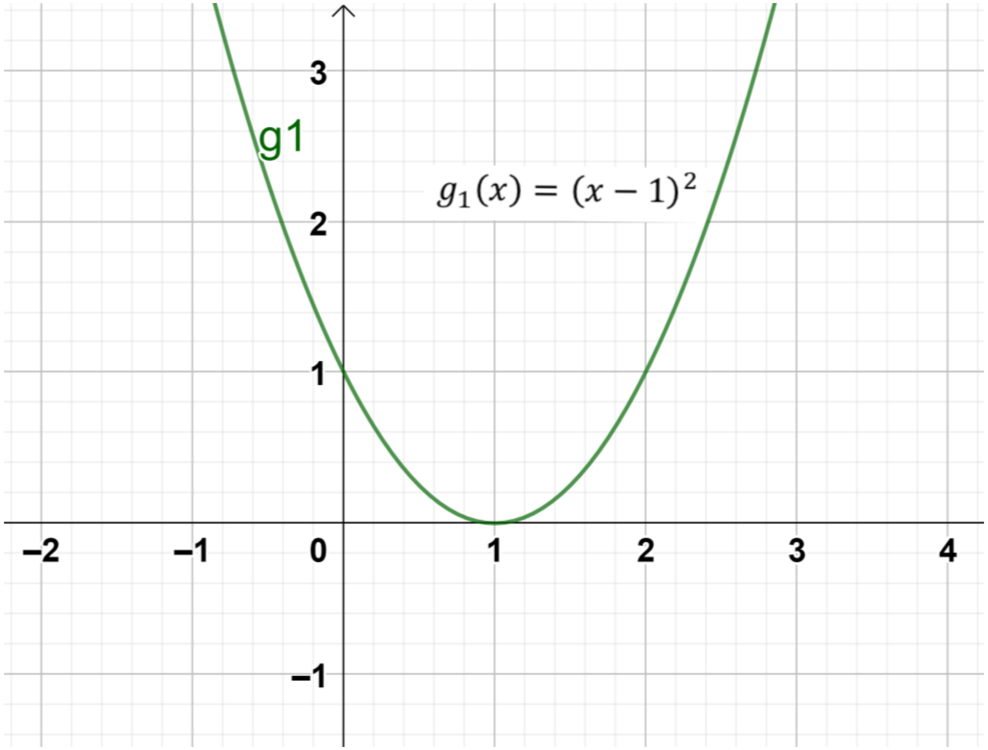


Vertical and Horizontal Stretching and Reflecting (5/6):

Starting with $f(x) = x^2$ to graph $g(x) = (x-1)^2 + 2$

$$f(x) = x^2$$

$$g_1(x) = (x-1)^2$$



Transformations of Funs. (2/2)

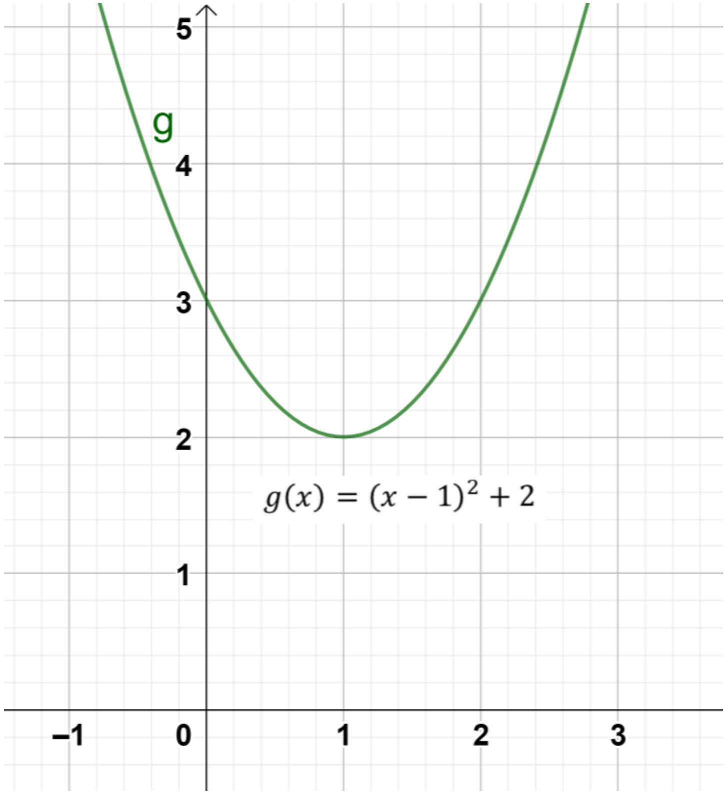
Vertical and Horizontal Stretching and Reflecting (5/6):

Starting with $f(x) = x^2$ to graph $g(x) = (x-1)^2 + 2$

$$f(x) = x^2$$

$$g_1(x) = (x-1)^2$$

$$g(x) = (x-1)^2 + 2$$

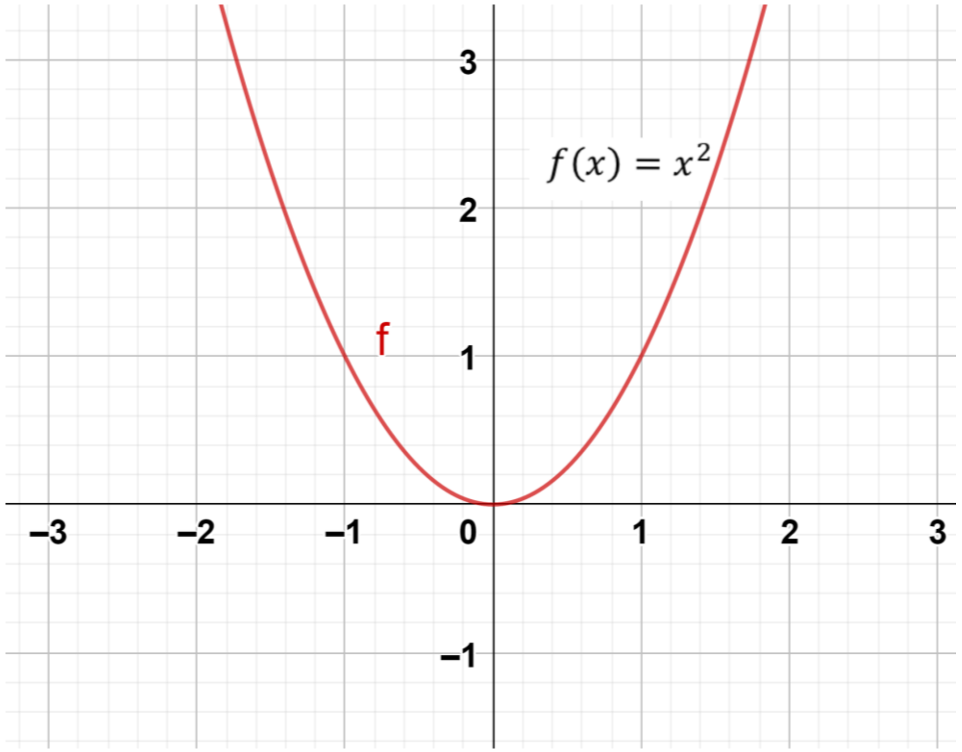


Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (6/6):

Starting with $f(x) = x^2$ to graph $g(x) = 1 - (x + 3)^2$

$$f(x) = x^2$$



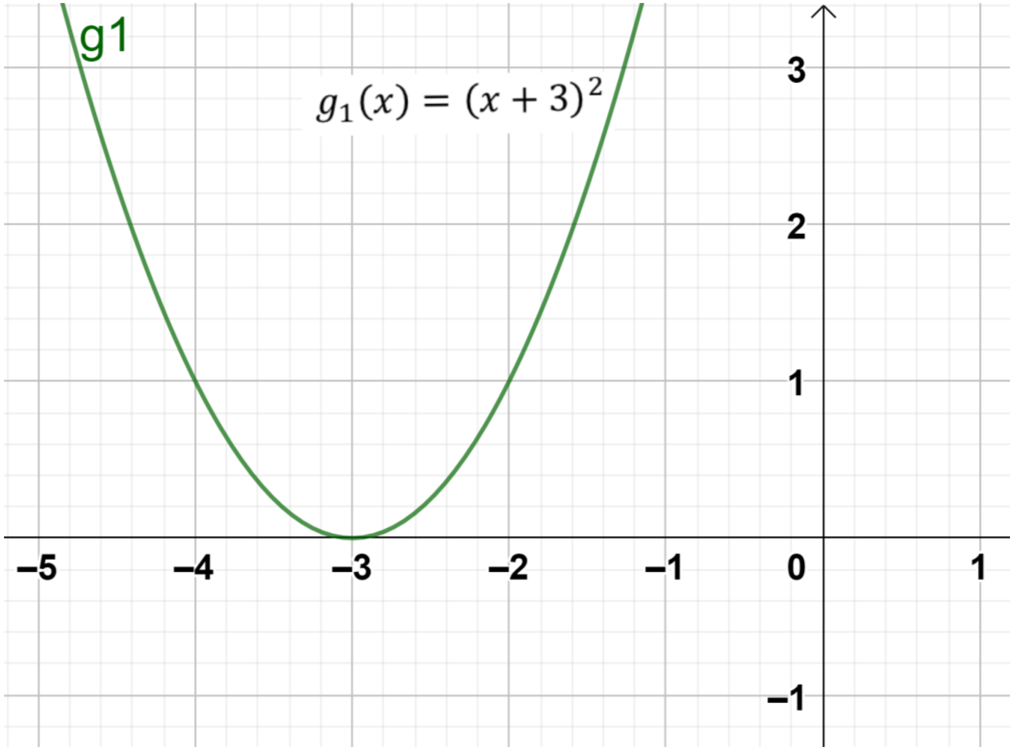
Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (6/6):

Starting with $f(x) = x^2$ to graph $g(x) = 1 - (x+3)^2$

$$f(x) = x^2$$

$$g_1(x) = (x+3)^2$$



Transformations of Funs. (2/2)

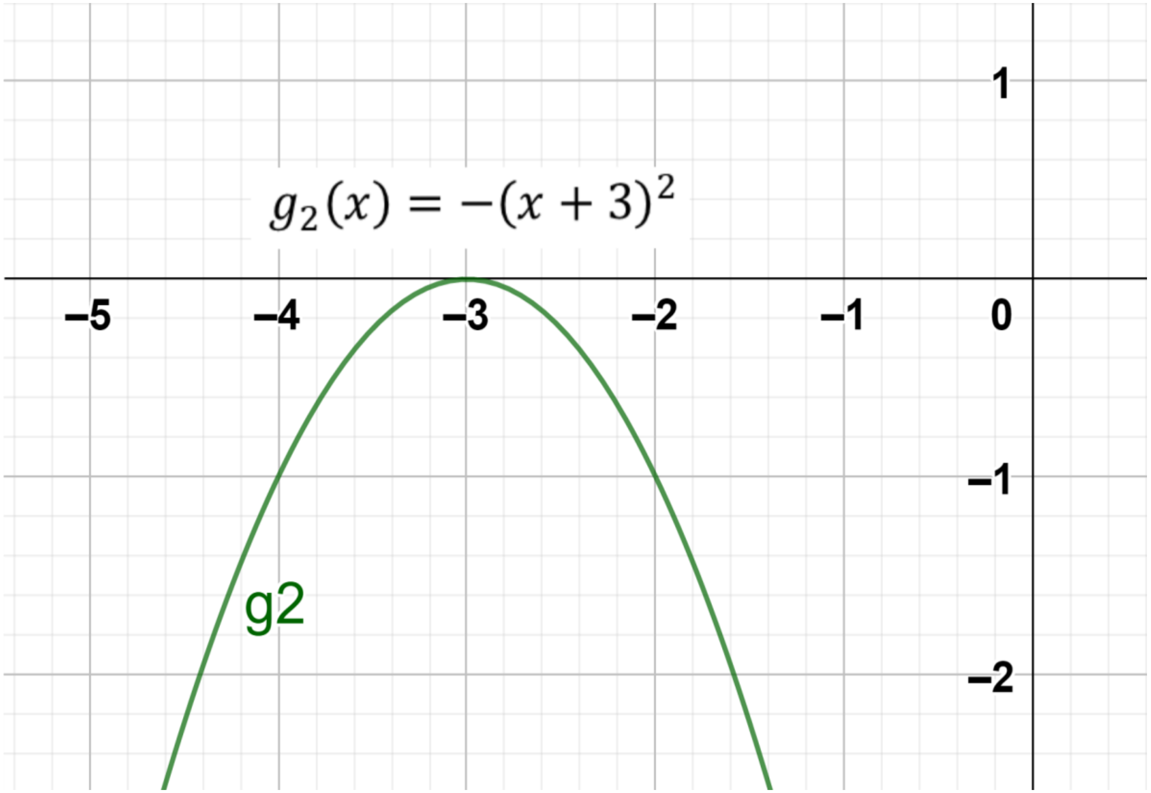
Vertical and Horizontal Stretching and Reflecting (6/6):

Starting with $f(x) = x^2$ to graph $g(x) = 1 - (x+3)^2$

$$f(x) = x^2$$

$$g_1(x) = (x+3)^2$$

$$g_2(x) = -(x+3)^2$$



Transformations of Funs. (2/2)

Vertical and Horizontal Stretching and Reflecting (6/6):

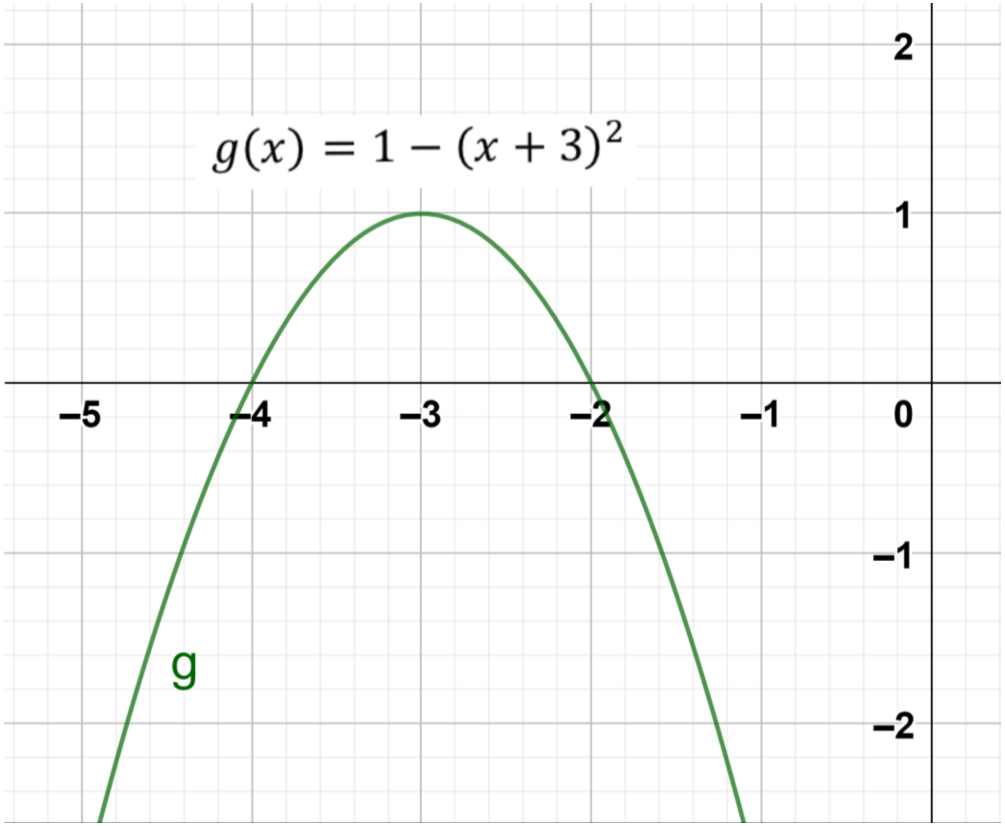
Starting with $f(x) = x^2$ to graph $g(x) = 1 - (x+3)^2$

$$f(x) = x^2$$

$$g_1(x) = (x+3)^2$$

$$g_2(x) = -(x+3)^2$$

$$g(x) = 1 - (x+3)^2$$



Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gkSI_PPAVJpebKDLo-ijEC

Lecture #3: https://www.youtube.com/watch?v=y8BYQmVM-zo&list=PLxIvc-MGOs6gkSI_PPAVJpebKDLo-ijEC&index=4

Thank You

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